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Blueprint for a dynamic deontic logic

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ABSTRACT

Extending the idiom of dynamic logic we outline a deontic logic in which deontic operators operate on terms rather than on formulæ. In a second step we distinguish between what we call real and deontic actions.

A famous paper by Alchourrón, Gärdenfors and Makinson opened up a new avenue of research into the logic of belief and belief change [1]. One of the later extensions is dynamic doxastic logic (DDL), which develops the AGM approach as a modal logic [7,12]. Work in this area continues.

It is noteworthy that the erstwhile interest in theory change of at least one of the founding fathers of AGM was not in belief change but in normative change. What the late Carlos Alchourrón, professor of jurisprudence, had originally wanted was, it seems, a logic of norms and norm change. Many years later it makes sense to ask whether there is a dynamic deontic logic (DΔL) that pursues Alchourrón's ambition. In this note we outline a blueprint of an answer. (For previous efforts by this author to provide such an answer, see in particular [11,15] and [16].)

We proceed in three steps. Already Georg Henrik von Wright, the founder of modern deontic logic, came to the conclusion that deontic logic must be built on a logic of action. In accordance with this insight we outline in Section 1 a logic of action, admittedly fairly meagre: it avoids a number of important but difficult topics, such as agency, causality and intentionality. In Section 2 we outline a deontic logic which is dynamic in the sense of allowing for what we call real actions. In Section 3 we suggest how one can also provide for what we call deontic actions.

In Appendix A three well-known paradoxes of deontic logic are discussed in the light of the theory developed in Section 2.

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[1] Some of the work reported in this note was carried out when the author was a fellow-in-residence in 2006 at N.I.A.S., the Netherlands Institute of Advanced Study at Wassenaar. An earlier version of the paper was presented in 2007 at a workshop on dynamic logic at the Université du Québec à Montréal as well as at a Dagstuhl Seminar. The paper was revised during the tenure of a Killam Visiting Fellowship at the University of Calgary 2008-9. The author wishes to thank two anonymous referees for helpful comments, and students and faculty in the Calgary philosophy department for stimulating discussions. Special thanks are also due to Olivier Roy.

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1. A temporal logic of action

1.1. Model theory

Without giving rigorous explanations, let us outline some key concepts. The base of any model will be a set (universe) U of points called the *environment*. Nonempty sequences of points will be called *paths*; they can be either finite or infinite in one or two directions. One-element sequences are regarded as *trivial*. A path p is a *subpath* of a path q if there are paths r and s such that $q = rps$; if r is trivial, p is a an *initial subpath* (is an *initial*) of q, while if s is trivial, q is a *final subpath* (is *final* in) q. Two paths p and q can be combined into one path, denoted by pq, if p has a last element $p(\#)$ and q has a first element $q(*)$ and the two are the same (if not, we regard the notation pq as meaningless). Notice that if $p(*) = u$ and $p(\#) = v$, then $\langle u, u\rangle p = p = p\langle v, v\rangle$.

Another fundamental model theoretical ingredient is that of a given set E of *actions* or *events* in U.[2] An *event* in U is a set of finite paths in U. If a is an event and $p \in a$, we say that *p realizes a* (p is a *realization* of a). Two special events are the *trivial* event, which is realized by every one-element sequence, and the *impossible* event, which is never realized. One can think of a number of set-theoretical operations on events under which E should be closed, for example, the sum $a \cup b$, the relative product $a \mid b$ and the difference $a - b$. Other set-theoretical operations on events, more difficult to relate to informal intuitions, are the product, the universal complement and Kleene's star operation, and they are not considered here. (But see [2] for one effort to accommodate the complement of events.)

Yet another fundamental concept is a given set H of *(complete) histories* in U—paths in U that are complete in the sense that if f is a proper subpath of a history h, then f is not itself a history. If a history is of the form hg, where thus the last element $h(\#)$ of h is also the first element $g(*)$ of g, then we will refer to (h, g) as an *articulated history*. One may say that (h, g) represents a particular way of looking at hg with h as the past, g as the future and the point $h(\#) = g(*)$ as the present.

We say that h is a *(possible) past* if $hg \in H$ for some g, while g is a *(possible) future* if $hg \in H$ for some h. If h is a past, then we write $\mathrm{cont}(h)$ for the set $\{g\colon\ hg \in H\}$ of possible continuations (possible futures) of h. Our pasts and futures are nonstrict in the sense that they include the present. In particular, if h is a past and g is a future that is trivial in the sense that $hg = h$, then the present is represented by a unique element u such that $u = h(\#)$ and $g = \{\langle u, u\rangle\}$.

If $S \subseteq \mathrm{cont}(h)$ we refer to (h, S) as a *possible situation* (with S the set of possible futures). In the special case that $S = \mathrm{cont}(h)$ we say that $(h, S) = (h, \mathrm{cont}(h))$ is an *actual situation*. Furthermore, if (h, S) is a possible situation, and if p is a finite path such that $h(\#) = p(*)$, then we write $S^p = \{f\colon\ pf \in S\}$. And if (h, S) is a possible situation and $g \in S$ we say that (h, g, S) is a *possible scenario* (note that in this case hg is a complete history).

If f is a history or a past or a future we say that f *includes* an event a if f contains a subpath that realizes a, and that *f excludes a* if there is no such subpath.

1.2. Syntax and meaning conditions

Our object languages must contain a denumerable set of propositional letters (primitive formulæ) $\mathsf{P}_0, \mathsf{P}_1, \ldots, \mathsf{P}_n, \ldots$ and a disjoint denumerable set $\mathsf{e}_0, \mathsf{e}_1, \ldots, \mathsf{e}_n, \ldots$ of event letters (primitive terms). In addition there has to be an adequate supply of Boolean (truth-functional) connectives as well as special operators; the latter will include at least the sum operator (+), and we will very briefly consider also two concatenation operators (; and ;;). Whatever the details, our language will contain both formulæ and terms.

A *basic frame* is a triple (U, E, H) such that U is a universe, E is a set of events (with certain closure conditions) and H is a set of complete histories. A *valuation* is a function V from the set of propositional letters into the power set of U and from the set of event letters into E. This function is extended in a natural way to all pure Boolean formulæ and to all terms. We will write $[\![\phi]\!]$ for the value assigned to a pure Boolean formula ϕ and $[\![\alpha]\!]$ for the value assigned to a term α. Thus, for all n,

$$[\![\mathsf{e}_n]\!] = V(\mathsf{e}_n),$$

$$[\![\mathsf{P}_n]\!] = V(\mathsf{P}_n).$$

Examples of meaning conditions (where ϕ and ψ are pure Boolean formulæ, and α and β are terms):

$$[\![\neg\phi]\!] = U - [\![\phi]\!],$$

$$[\![\phi \wedge \psi]\!] = [\![\phi]\!] \cap [\![\psi]\!],$$

$$[\![\phi \vee \psi]\!] = [\![\phi]\!] \cup [\![\psi]\!],$$

$$[\![\alpha + \beta]\!] = [\![\alpha]\!] \cup [\![\beta]\!],$$

[2] Many philosophers distinguish between actions and events, as they should. However, in this paper we do not.