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Similarity based approximate reasoning: fuzzy control

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Abstract

This paper presents an approach to similarity based approximate reasoning that elucidates the connection between similarity and existing approaches to inference in approximate reasoning methodology. A set of axioms is proposed to get a reasonable measure of similarity between two fuzzy sets. The similarity between the fact(s) and the antecedent of a rule is used to modify the relation between the antecedent and the consequent of the rule. An inference is drawn using the well-known projection operation on the domain of the consequent. Zadeh's compositional rule of inference and existing similarity based reasoning techniques are considered for a new similarity based approximate reasoning technique. The proposed mechanism is used to develop a modified fuzzy control system. A new defuzzification scheme is proposed. Simulation results are presented for the well-known inverted pendulum problem.

Keywords: Approximate reasoning; Fuzzy set; Similarity index; Specificity measure; Fuzzy control

1. Introduction

The cognitive process of human reasoning deals with imprecise premises. Traditional two-valued logic and/or multi-valued logics are not effective in handling such reasoning processes. Zadeh developed approximate reasoning methodology to tackle the complex problem [40,42]. The desire to build up a quantitative framework that will allow us to derive an approximate conclusion from imprecise knowledge is the main motivation of the theory of approximate reasoning. Fuzzy logic is the basis of approximate reasoning. A proposition in fuzzy logic is represented by a fuzzy set [38].

A collection of imprecise information given by human experts often forms the basis of a fuzzy system which is represented by fuzzy sets or fuzzy relations. The task of a fuzzy system is to exploit the knowledge acquired by experts over time and model the world with it. A fuzzy system reasons with its knowledge. Different patterns of reasoning in human beings indicate a need for similarity matching, in situations where there is no directly applicable knowledge, to come up with a plausible conclusion. In such cases, the confidence in a conclusion may be determined, based on a degree of similarity between the fact(s) and the antecedent of a rule. We know that fuzzy set theory is based on the fuzzification of the predicate 'belongs to'. The indistinguishability modelled by fuzzy set is computable and this concept cannot be overcome in approximate reasoning [11]. In order to capture this, an inference model should have the required flexibility. Specifically, we need means to handle graded information on one hand and the concept of

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similarity on the other hand. Conventional approximate reasoning does not consider the concept of similarity measure in deriving a consequence. Existing similarity based reasoning methods modify the consequence part of a rule, based on a measure of similarity and therefore, the consequence becomes independent of the conditionals. To satisfy both the requirements simultaneously, we need to integrate conventional approximate reasoning and similarity based reasoning for an adequate theory of similarity based approximate reasoning.

Zadeh introduced the concept of Compositional Rule of Inference (CRI). Let us consider Zadeh's form of inference in approximate reasoning based on the Compositional Rule of Inference: From 'X is A' and '(X, Y) is R' infer 'Y is B'. Symbolically, $B = A \circ R$ which is explicitly given by

$$\mu_B(v) = \sup_{u \in U} \{ T(\mu_A(u), \mu_R(u, v)) \}; \tag{1}$$

where A is a fuzzy subset of U, B is a fuzzy subset of V and R is a fuzzy binary relation on $U \times V$, U and V being the universes of discourse of the linguistic variables X and Y, T is a T-norm function. CRI scheme is such that for a large class of A, each different from the other, the concluded B remains the same (since we are using sup/inf type of operation). Such relations may also produce significant conclusions from an almost dissimilar pair $\{A, A^*\}$ (it is easy to see that whatever A^* be we can always derive a conclusion using CRI). In [31], the authors proposed an alternative model for similarity based analogical approximate reasoning. Recently in [30], a similar scheme for similarity based reasoning has been propounded. In similarity based reasoning, from a given fact the conclusion is derived based on a measure of similarity between the fact and the antecedent of a rule. As for example, let $U = \{a, b, c, d\}$ and $V = \{u, v, w, x\}$ be the universes of discourse,

$$A = \{(1.0, a), (0.75, b), (0.5, c), (0.25, d)\} \quad \text{and}$$

$R =$

	u	v	w	x
a	1.00	0.75	0.50	0.25
b	0.75	1.00	0.75	0.50
c	0.50	0.75	1.00	0.75
d	0.25	0.50	0.75	1.00

then taking $T = \min$ and using CRI we find $B = \{(1.00, u), (0.75, v), (0.75, w), (0.50, x)\}$.

This shows that the linguistic variables X and Y are approximately equal. A careful scrutiny of the relation also says so. It is easy to see that the conclusion B will remain the same if we choose $A' = \{(1.0, a), (0.75, b)\}$ or $A' = \{(1.0, a), (0.75, c), (0.50, d)\}$, which are highly dissimilar to A. Next, if we take $A = \{(1.0, a)\}$ then from R we have $B = \{(1.00, u), (0.75, v), (0.50, w), (0.25, x)\}$ again, if we take $A = \{(1.0, d)\}$ then $B = \{(0.25, u), (0.50, v), (0.75, w), (1.00, x)\}$. This shows that even if the input values are strongly complementary to each other, significant conclusions can be drawn using Zadeh's CRI.

In a rule based system, in general, we deduce a conclusion B' from and observed fact A' and a general rule $A \to B$ using Sup-T composition as

$$\mu_{B'}(v) = \sup_{u \in U} \{ T(\mu_{A'}(u), \mu_R(u, v)) \} \tag{2}$$

where $\mu_R(u, v) = T_1(\mu_A(u), \mu_B(v))$. If we choose, for convenience, $T = T_1 = \min$ then from $A = \{(1.0, a), (0.75, b), (0.5, c), (0.25, d)\}$, $B = \{(1.0, a), (0.75, b), (0.5, c), (0.25, d)\}$ and $A' = \{(1.0, a), (0.5625, b), (0.25, c), (0.0625, d)\}$ we find that

$R =$

	u	v	w	x
a	1.00	0.75	0.50	0.25
b	0.75	0.75	0.50	0.25
c	0.50	0.50	0.50	0.25
d	0.25	0.25	0.25	0.25

and that $B' = B$. If, instead, we choose $A' = \{(1.00/a)\}$ then also we find that $B' = B$.

In similarity based reasoning we consider the statements

$$p: \text{ if } X \text{ is } A \text{ then } Y \text{ is } B, \tau \text{ and } q : X \text{ is } A'.$$