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GLOBAL SOLUTION TO 1D MODEL OF A COMPRESSIBLE VISCOUS MICROPOLAR HEAT-CONDUCTING FLUID WITH A FREE BOUNDARY*

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Abstract In this paper we consider the nonstationary 1D flow of the compressible viscous and heat-conducting micropolar fluid, assuming that it is in the thermodynamically sense perfect and polytropic. The fluid is between a static solid wall and a free boundary connected to a vacuum state. We take the homogeneous boundary conditions for velocity, microrotation and heat flux on the solid border and that the normal stress, heat flux and microrotation are equal to zero on the free boundary. The proof of the global existence of the solution is based on a limit procedure. We define the finite difference approximate equations system and construct the sequence of approximate solutions that converges to the solution of our problem globally in time.

Key words micropolar fluid flow; initial-boundary value problem; free boundary; finite difference approximations; strong and weak convergence

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1 Introduction

This paper analyzes the compressible flow of the isotropic, viscous and heat conducting micropolar fluid, which is in the thermodynamical sense perfect and polytropic. The model for this type of flow was first considered by Mujaković in [1] where she developed a one-dimensional model. In the same work, the local existence and the uniqueness of the solution, which is called generalized, for our model with the homogeneous boundary conditions for velocity, microrotation and heat flux were proved. In the proof of this result the Faedo-Galerkin method was used. In the articles [2] and [3] the same method was used to prove the local existence theorems.

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In this paper we consider the problem for the same fluid placed between a static solid wall and a free boundary connected to a vacuum state. We set up our problem first in the Euler variables (y, t). Let at the initial moment $t = 0$ fluid occupy a bounded domain $[0, L]$. Let the left boundary $y = 0$ be an impermeable solid wall and the right one $y = y_0(t)$ be a boundary where fluid contacts with vacuum and be unknown in advance. We also assume the homogeneous boundary conditions for velocity, microrotation and heat flux on the fixed border and the homogeneous boundary conditions for the strain, microrotation and heat flux on the free boundary. Under the transition to the Lagrange coordinates (x, t) (see [1]) we get the same boundary conditions at the borders $x = 0$ and $x = 1$. Here we prove the global existence of the solution for the described problem. In the proof, we do not use as in [1], [2] and [3] the Faedo-Galerkin method, which is not applicable to this type of problem. We use the finite difference method instead. We applied the same method in [4] where the existence of global solution for the problem with homogeneous boundary conditions, defined in [1], was proved. In this paper the proof of the global solution is based on a similar procedure as in [4]. Unlike in [4], where the positive lower and upper bounds for mass density existed, the main difficulty here was the nonexistence of the positive lower bound for the mass density due to the assumed free boundary condition. As a consequence of that, in this paper more complicated estimating procedure than in [4], was necessary.

Let notice that in all previous research it was enough to assume that the initial density belongs to $H^1(\langle 0, 1\rangle)$. In order to obtain more precise estimates, we propose here that it belongs to $H^2(\langle 0, 1\rangle)$. In this paper we use some ideas from [5] where a similar method was used to prove the existence of the global solution for the classical fluid flow problem with spherical symmetry. However, the microrotation velocity that appears in our model causes additional difficulties in investigating the existence of the solution for the considered problem.

In our work, the proof is based on a limit procedure. We define the finite difference approximate equations system, investigate the properties of the sequence of the approximate solutions and prove that the limit of this sequence is the solution to our problem on the domain $(0, 1) \times (0, T)$, where $T > 0$ is arbitrary. We follow some ideas of [6, 7] also.

The paper is organized as follows. In Section 2 we introduce the mathematical formulation of our problem. In Section 3 we derive the finite difference approximate equations system and in the fourth section present the main result. In Sections 5–8, we prove uniform a priori estimates for the approximate solutions. Finally, the proof of convergence of a sequence of approximate solutions to a solution of our problem is given in the ninth section.

2 Mathematical Model

We are dealing with the one-dimensional flow of the compressible viscous and heat-conducting micropolar fluid flow, which is thermodynamically perfect and polytropic. Let ρ, v, w and θ denote, respectively, the mass density, velocity, microrotation velocity and temperature in the Lagrangian description. The motion of the fluid under consideration is described by the following system of four equations (see [1]):

$$\partial_t \rho + \rho^2 \partial_x v = 0, \tag{2.1}$$

$$\partial_t v = \partial_x\big(\rho\, \partial_x v\big) - K \partial_x(\rho\, \theta), \tag{2.2}$$