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A PROJECTION-TYPE ALGORITHM FOR SOLVING GENERALIZED MIXED VARIATIONAL INEQUALITIES*

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Abstract We propose a projection-type algorithm for generalized mixed variational inequality problem in Euclidean space $\mathbb{R}^n$. We establish the convergence theorem for the proposed algorithm, provided the multi-valued mapping is continuous and f-pseudomonotone with nonempty compact convex values on $\mathrm{dom}(f)$, where $f:\mathbb{R}^n\to\mathbb{R}\cup\{+\infty\}$ is a proper function. The algorithm presented in this paper generalize and improve some known algorithms in literatures. Preliminary computational experience is also reported.

Key words projection-type algorithm; generalized mixed variational inequality; f-pseudomonotone mapping

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1 Introduction

Let $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$ denote the usual inner product and norm in $\mathbb{R}^n$, respectively. Let $f:\mathbb{R}^n\to\mathbb{R}\cup\{+\infty\}$ be a proper convex lower semicontinuous function and $F:\mathbb{R}^n\to 2^{\mathbb{R}^n}$ be a multi-valued mapping. In this paper, we consider the generalized mixed variational inequality problem, denoted by GMVI(F, f, dom(f)), which be defined as

$$\text{Find } x\in\mathrm{dom}(f) \text{ and } \xi\in F(x) \text{ such that } \langle\xi, y-x\rangle+f(y)-f(x)\geq 0,\ \ \forall y\in\mathrm{dom}(f), \quad (1.1)$$

where $\mathrm{dom}(f)=\{x\in\mathbb{R}^n: f(x)<+\infty\}$ denotes the effective domain of f. Let S be the solution set of problem (1.1).

Problem (1.1) is encountered in many applications, in particular in mechanical problems and equilibrium problems, and hence numerical methods and formulations are studied, see [1–9]. On the other hand, the generalized mixed variational inequality problem GMVI(F, f, dom(f)) includes a large variety of problems as special instance. For example, let F be the subdifferential of a finite-valued convex continuous function ψ defined on $\mathbb{R}^n$, the GMVI(F, f,

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dom(f)) becomes the following unconstraint convex optimization problem

$$\min_{x\in\mathbb{R}^n}\{f(x)+\psi(x)\}.$$

We remark that if F be a single-valued mapping, the problem (1.1) is equivalent to the mixed variational inequality problem

$$\text{Find } x^*\in \mathrm{dom}(f) \text{ such that } \langle F(x^*), y-x^*\rangle + f(y) - f(x^*) \geq 0, \quad \forall y\in \mathrm{dom}(f). \tag{1.2}$$

A number of papers in the literature studied the theoretical properties and solution algorithms of problem (1.2), see [10–15]. Recently, He [13] extended Algorithm 2.1 of [16] to solve the mixed variational inequality problem. The iteration sequence generated by the algorithm converges to a solution, provided F is f-pseudomonotone and continuous on dom(f), and f is Lipschitz continuous on dom(f).

Furthermore, if f be the indicator function of a nonempty closed convex set $K\subset\mathbb{R}^n$, that is,

$$I_K(x)=\begin{cases}0, & \text{if } x\in K;\\ +\infty, & \text{otherwise.}\end{cases}$$

Then problem (1.2) reduces to the variational inequality problem (in short, VI(F,K)):

$$\text{find } x^*\in K \text{ such that } \langle F(x^*), y-x^*\rangle \geq 0, \quad \forall y\in K. \tag{1.3}$$

Many algorithms for solving the VI(F,K) are projection algorithms that employ projections onto the feasible set K of VI(F,K), or onto some related sets, in order to iteratively reach a solution, see [16–20] and the references therein. In particular, Solodov and Svaiter [16] suggested a new projection method, known as the double projection method, for solving problem (1.3). It consists two steps. First, a hyperplane is constructed, which strictly separates current iterate from the solution set. The construction of this hyperplane requires an Armijo-type linesearch. Then the next iterate is produced by projecting the current iterate onto the intersection of the set K and the hyperplane. [18] showed that Solodov and Svaiter's method obtain a longer stepsize, and guarantee that the distance from the next iterative point to the solution set has a large decrease; and hence it's a better method than the method proposed by Iusem and Svaiter [19].

On the other hand, if $f(x)=I_K(x)$ for all $x\in\mathbb{R}^n$, then problem (1.1) collapses to:

$$\text{find } x^*\in K \text{ and } \xi^*\in F(x^*) \text{ such that } \langle \xi^*, y-x^*\rangle \geq 0, \quad \forall y\in K, \tag{1.4}$$

which is called the classical generalized variational inequality problem, denoted by GVI(F,K). Browder [21] introduced problem (1.4) and studied the existence of its solution. Since then, theory and algorithm of GVI(F,K) were much studied in the literatures, see [1, 17, 21–25] and the references therein. By using different Armijo-type linesearches and constructing profitable hyperplanes, which separating strictly current point x_i and the solution set S, [24, 25] proposed some double projection algorithms for generalized variational inequality problems. For these papers, the separating hyperplane determines the convergence speed of the sequence generated by the double projection algorithm.

Inspired and motivated by the above results, we suggest a new projection-type algorithm to solve the generalized mixed variational inequality problem. In our algorithm, we suggest a new