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STRONGLY CONVERGENT ITERATIVE METHODS FOR SPLIT EQUALITY VARIATIONAL INCLUSION PROBLEMS IN BANACH SPACES*

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Abstract The purpose of this paper is to introduce and study the split equality variational inclusion problems in the setting of Banach spaces. For solving this kind of problems, some new iterative algorithms are proposed. Under suitable conditions, some strong convergence theorems for the sequences generated by the proposed algorithm are proved. As applications, we shall utilize the results presented in the paper to study the split equality feasibility problems in Banach spaces and the split equality equilibrium problem in Banach spaces. The results presented in the paper are new.

Key words the split equality variational inclusion problem in Banach space; split feasibility problem in Banach space; split equilibrium problem in Banach spaces

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1 Introduction

Let C and Q be nonempty closed and convex subsets of real Hilbert spaces H_1 and H_2, respectively. The split feasibility problem (SFP) is formulated as

$$\text{to find } \ x^* \in C \quad \text{and} \quad Ax^* \in Q \tag{1.1}$$

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where $A : H_1 \to H_2$ is a bounded linear operator. In 1994, Censor and Elfving [1] first introduced the (SFP) in finite-dimensional Hilbert spaces for modeling inverse problems which arise from phase retrievals and in medical image reconstruction [2]. It was found that the (SFP) can also be used in various disciplines such as image restoration, computer tomograph and radiation therapy treatment planning [3–5]. The (SFP) in an infinite dimensional real Hilbert space can be found in [2, 4, 6–10].

Recently, Moudafi [11–13] introduced the following split equality feasibility problem (SEFP):

$$\text{to find} \quad x \in C, \quad y \in Q \quad \text{such that} \quad Ax = By, \tag{1.2}$$

where $A : H_1 \to H_3$ and $B : H_2 \to H_3$ are two bounded linear operators. Obviously, if $B = I$ (identity mapping on H_2) and $H_3 = H_2$, then (1.2) reduces to (1.1). The kind of split equality feasibility problems (1.2) allows asymmetric and partial relations between the variables x and y. The interest is to cover many situations, such as decomposition methods for PDEs, applications in game theory and intensity-modulated radiation therapy.

In order to solve split equality feasibility problem (1.2), Moudafi [11] introduced the following simultaneous iterative method

$$\begin{cases} x_{n+1} = P_C(x_n - \gamma A^*(Ax_n - By_n)), \\ y_{n+1} = P_Q(y_n + \beta B^*(Ax_{n+1} - By_n)) \end{cases} \tag{1.3}$$

and under suitable conditions he proved the weak convergence of the sequence $\{(x_n, y_n)\}$ to a solution of (1.2) in Hilbert spaces.

Attempt to introduce and consider the split feasibility problem and split common null point problem in the setting of Banach spaces have recently been made. In 2015, Takahashi [14] first introduced and considered such problems in Banach spaces. By using hybrid methods and Halpern's type methods and under suitable conditions some strong and weak convergence theorems for such problems are proved in Banach spaces. The results presented in [14] seem to be the first outside Hilbert space.

Motivated by the above works and related literatures, the purpose of this paper is to introduce and study the following split equality variational inclusion problems in the setting of Banach spaces.

Let H_1 and H_2 be two real Hilbert spaces and F be a real Banach space. Let $A : H_1 \to F$, $B : H_2 \to F$ be two bounded linear operators and A^* and B^* be the adjoint mappings of A and B, respectively. In the sequel we always denote by $F(K)$ the fixed point set of a mapping K. Let $U_i : H_i \to 2^{H_i},\ i = 1, 2$ be a maximal monotone mapping. The resolvent of U_i is defined by

$$J_\mu^{U_i} = (I + \mu U_i)^{-1} : H_i \to H_i,\ \mu > 0,\ i = 1, 2.$$

It is easy to know that if $U_i : H_i \to 2^{H_i},\ i = 1, 2$ is a maximal monotone mapping, then the resolvent $J_\mu^{U_i}$ of U_i is a nonexpansive mapping and $F(J_\mu^{U_i}) = U_i^{-1}(0)$, where $U_i^{-1}(0)$ is the set of zero points of U_i in H_i.

The "so-called" split equality variational inclusion problems in Banach spaces (SEVIP) is to find

$$p \in U_1^{-1}(0),\ q \in U_2^{-1}(0) \ \text{ such that } \ Ap = Bq. \tag{1.4}$$