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VANISHING VISCOSITY FOR NON-ISENTROPIC GAS DYNAMICS WITH INTERACTING SHOCKS*

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Abstract In this paper, we study the vanishing viscosity limit for non-isentropic gas dynamics with interacting shocks. Given any entropy solution of non-isentropic gas dynamics which consists of two different families of shocks interacting at some positive time, we show that such solution is the vanishing viscosity limit of a family of smooth global solutions for a viscous system of conservation law. We remark that, after the interacting time, not only shocks but also contact discontinuity are generated.

Key words vanishing viscosity; non-isentropic gas dynamics; interacting shock; contact discontinuity

2010 MR Subject Classification 35Q31; 35B65

1 Introduction and Main Results

In this paper, we consider the vanishing viscosity limit of solutions to non-isentropic gas dynamics with artificial viscosity

$$\begin{cases} v_t - u_x = \epsilon v_{xx}, \\ u_t + p_x = \epsilon u_{xx}, \\ E_t + (up)_x = \epsilon E_{xx}, \end{cases} \tag{1.1}$$

where $v(x,t) > 0$ is the special volume, $u(x,t)$ is the velocity, and $\theta > 0$ is the absolute temperature of the gas. The pressure p and the total energy E are given by the equations of

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state

$$p=\frac{R\theta}{v} \quad \text{and} \quad E=\frac{R\theta}{\gamma-1}+\frac{u^2}{2},$$

where the constants $R>0$ and $\gamma>1$ denote the gas constant and the adiabatic exponent, respectively.

Formally, as $\epsilon\to 0^+$, system (1.1) tends to the following non-isentropic gas dynamics system

$$\begin{cases} v_t-u_x=0,\\ u_t+p_x=0,\\ E_t+(up)_x=0. \end{cases} \tag{1.2}$$

The vanishing viscosity limit of solutions to gas dynamics system was an open and challenging problem with a long history. When the solution of gas dynamics system is smooth, the problem can be studied by scaling method. However, it is known that its solution in general develops singularity in the finite time such as shock waves. Thus it is interesting but difficult to study the vanishing viscosity in the case when the solution of gas dynamics system does not possess smooth solution.

Now let us give some related works on the vanishing viscosity limit of viscous fluid system. For general hyperbolic conservation laws with artificial viscosity

$$\mathbf{u}_t^\epsilon+\mathbf{f}(\mathbf{u}^\epsilon)_x=\epsilon\mathbf{u}_{xx}^\epsilon, \tag{1.3}$$

Goodman and Xin [5] proved the vanishing viscosity limit for (1.3) in L^∞ norm for piecewise smooth solutions separated by non-interacting shocks. For the Navier-Stokes equations, i.e., the viscosity matrix is not positive definite, Hoff-Liu [6], Xin [25] proved the vanishing viscosity limit of single shock and rarefaction wave, respectively. We refer to [7–12, 14–16, 19, 24, 27, 28] and reference therein for related works. For interacting shocks, Serre [21] showed the vanishing viscosity limit associated with general viscous conservation laws in L^2 norm. Bianchini-Bressan [1], Bressan-Huang-Wang-Yang [2] and Bressan-Yang [3] studied the vanishing viscosity limit in BV space. Huang-Wang-Wang-Yang [13] first studied the vanishing viscosity limit for one-dimensional isentropic compressible Navier-Stokes equations in the case of two different of families of interacting shocks in L^∞ norm which gives detailed information of vanishing viscosity limits. Precisely speaking, they studied the simplest case, where after the shocks interact, only shocks are generated. Moreover, they pointed out that it is interesting and meaningful to study the problem of the vanishing viscosity of non-isentropic system for the interacting shocks. Along this direction, we begin to study the vanishing viscosity problem for (1.1) with artificial viscosity. It is noted even in the simplest case where the two shocks belong to two different of families, not only two outgoing shocks but also a contact discontinuity are generated after the two shocks hit. This makes the problem complicated. The vanishing viscosity limit for the non-isentropic Navier-Stokes equations is difficult and left for the future.

Now let us formulate our main result. Note that the eigenvalues of the non-isentropic gas dynamics system (1.2) are

$$\lambda_1=-\sqrt{\frac{\gamma p}{v}}<0,\ \ \lambda_2=0,\ \ \lambda_3=\sqrt{\frac{\gamma p}{v}}>0, \tag{1.4}$$

where the first and the third characteristic fields are genuinely nonlinear and the second characteristic field is linearly degenerate.