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PROPERTIES OF THE MODIFIED ROPER-SUFFRIDGE EXTENSION OPERATORS ON REINHARDT DOMAINS*

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Abstract In this paper, we mainly discuss the properties of the modified Roper-Suffridge operators on Reinhardt domains. By the analytical characteristics and distortion results of subclasses of biholomorphic mappings, we conclude that the modified Roper-Suffridge operators preserve the properties of $S^*_\Omega(\beta, A, B)$, almost starlike mapping of complex order λ on $\Omega_{n,p_2,\cdots,p_n}$. Sequentially, we get that the modified Roper-Suffridge operators preserve spirallikeness of type β and order α, strongly pirallikeness of type β and order α, almost starlikeness of order α on $\Omega_{n,p_2,\cdots,p_n}$. The conclusions provide a new approach to construct these biholomorphic mappings which have special geometric properties in several complex variables.

Key words Roper-Suffridge operator; spirallike mappings; starlike mappings

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1 Introduction

The property of biholomorphic mappings is one of the most important objects of study in geometric function theory of several complex variables. Starlike mappings and convex mappings are the mappings discussed more. In recent years, many people discussed the subclasses or extensions of starlike mappings and convex mappings, such as almost spirallike mappings of order α and type β. It is easy to find specific examples of these new subclasses or extensions in one complex variable, which will enable us to better study these mappings. While, it is very difficult to find specific examples in several complex variables.

In 1995, Roper and Suffridge [1] introduced an operator

$$\phi_n(f)(z) = \Big(f(z_1), \sqrt{f'(z_1)}z_0\Big)',$$

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where $z=(z_1,z_0)\in B^n, z_1\in D, z_0=(z_2,\cdots,z_n)\in\mathbb{C}^{n-1}, f(z_1)\in H(D), \sqrt{f'(0)}=1$. Roper and Suffridge proved the Roper-Suffridge operator preserves convexity and starlikeness on B^n. Graham and Kohr [2] proved the Roper-Suffridge operator preserves the properties of Bloch mappings on B^n. Thus, through the Roper-Suffridge operator, we can construct lots of convex mappings and starlike mappings on B^n by corresponding functions on the unit disk D of $\mathbb{C}$. So the Roper-Suffridge operator plays an important role in several complex variables. Later, many people devoted themselves to the research of the Roper-Suffridge extension operator. They generalized the Roper-Suffridge operator on different domains and different spaces so as to construct biholomorphic mappings with specific geometric properties in several complex variables. In recent years, there were many results about the generalized Roper-Suffridge operators (see [3–5]).

In 2005, Muir [6] introduced the generalized Roper-Suffridge operator

$$F(z)=\Big(f(z_1)+f'(z_1)P(z_0),\sqrt{f'(z_1)}z_0\Big)',$$

where f is a normalized biholomorphic function on the unit disk D, $z=(z_1,z_0)'\in B^n, z_1\in D, z_0=(z_2,\cdots,z_n)'\in\mathbb{C}^{n-1}$. The branch of the power function is chosen such that $\sqrt{f'(0)}=1$. $P:\mathbb{C}^{n-1}\to\mathbb{C}$ is a homogeneous polynomial of degree 2. Muir and Suffridge [7] proved the extended operator preserves starlikeness and convexity on $\|P\|\le\frac{1}{4}$ and $\|P\|\le\frac{1}{2}$, respectively. Kohr and Muir [8, 9] discussed the extended operator by Loewner chains. Wang and Liu [10] discussed the extended Roper-Suffridge operator

$$F(z)=\Big(f(z_1)+f'(z_1)P(z_0),[f'(z_1)]^{\frac{1}{m}}z_0\Big)' \tag{1.1}$$

preserves almost starlikeness of order α and starlikeness of order α under some conditions on the unit ball B^n, where $[f'(0)]^{\frac{1}{m}}=1$ and $P:\mathbb{C}^{n-1}\to\mathbb{C}$ is a homogeneous polynomial of degree $m(m\in\mathbb{N},m\ge 2)$.

Muir [9] introduced the extended Roper-Suffridge operator

$$[\Phi_{G,\gamma}(f)](z)=(f(z_1)+G([f'(z_1)]^\gamma z_0),[f'(z_1)]^\gamma z_0)' \tag{1.2}$$

on the unit ball B^n in $\mathbb{C}^n$, where $z=(z_1,z_0)$ and $f(z_1)$ is a normalized univalent holomorphic function on D, G is a holomorphic function in $\mathbb{C}^{n-1}$ with $G(0)=0, DG(0)=I, \gamma\ge 0$ and $[f'(z_1)]^\gamma|_{z_1=0}=1$. The homogeneous expansion of $G(z)$ is $\sum\limits_{j=0}^{\infty}Q_j(z)$, where $Q_j(z)$ is a homogeneous polynomial of degree j. Obviously, (1.2) reduces to (1.1) if $\gamma=\frac{1}{m}$ and $j=m$. Muir proved $[\Phi_{G,\gamma}(f)](z)$ is a Loewner chain preserving extension operator provided that G satisfies some conditions.

In this paper, we mainly seek conditions under which the generalized operator (1.2) preserves the properties of subclasses of biholomorphic mappings. In Sections 2 and 3, we discuss (1.2) preserves the properties of $S^*_\Omega(\beta,A,B)$, almost starlike mapping of complex order λ on $\Omega_{n,p_2,\cdots,p_n}$ under different conditions, respectively. Thereby, we get that (1.2) preserves spirallikeness of type β and order α, strongly spirallikeness of type β and order α, almost starlikeness of order α on $\Omega_{n,p_2,\cdots,p_n}$. The conclusions generalize some known results.

In the following, let D denote the unit disk in $\mathbb{C}$. Let $J_F(z)$ denote the Fréchet derivative of F at z. Let $I[a/b]$ denote the integer part of $\frac{a}{b}$.

To get the main results, we need the following definitions and lemmas.