Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2016,**36B**(6):1780–1792
Acta Mathematica Scientia 数学物理学报
http://actams.wipm.ac.cn

EXISTENCE AND UNIQUENESS OF NON-TRIVIAL SOLUTION OF PARABOLIC p-LAPLACIAN-LIKE DIFFERENTIAL EQUATION WITH MIXED BOUNDARIES*

Li WEI (魏利) *Rui CHEN* (陈蕊)
School of Mathematics and Statistics, Hebei University of Economics and Business, Shijiazhuang 050061, China
E-mail: *diandianba@yahoo.com; stchenri@heuet.edu.cn*

Ravi P. AGARWAL
Department of Mathematics, Texas A&M University-Kingsville, Kingsville, TX 78363, USA;
Department of Mathematics, Faculty of Science, King Abdulaziz University, 21589 Jeddah, Saudi Arabia
E-mail: *Ravi.Agarwal@tamuk.edu*

Patricia YJ WONG
School of Electrical and Electronic Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798, Singapore
E-mail: *ejywong@ntu.edu.sg*

Abstract One parabolic p-Laplacian-like differential equation with mixed boundaries is studied in this paper, where the item $\frac{\partial u}{\partial t}$ in the corresponding studies is replaced by $\alpha(\frac{\partial u}{\partial t})$, which makes it more general. The sufficient condition of the existence and uniqueness of non-trivial solution in $L^2(0,T;L^2(\Omega))$ is presented by employing the techniques of splitting the boundary problems into operator equation. Compared to the corresponding work, the restrictions imposed on the equation are weaken and the proof technique is simplified. It can be regarded as the extension and complement of the previous work.

Key words maximal monotone operator; Caratheodory'conditions; subdifferential; p-Laplacian-like equation; nontrivial solution

2010 MR Subject Classification 47H05; 47H09

1 Introduction

Nonlinear boundary value problems (NBVPs) involving the p-Laplacian operator arise from many physical phenomena, such as reaction-diffusion problems, petroleum extraction, flow

*Received April 13, 2015; revised May 1, 2016. The first author is supported by the National Natural Science Foundation of China (11071053), Natural Science Foundation of Hebei Province (A2014207010), Key Project of Science and Research of Hebei Educational Department (ZD2016024) and Key Project of Science and Research of Hebei University of Economics and Business (2015KYZ03).

through porous media and non-Newtonian fluids. Thus, it is a hot topic to study such problems and their generalizations by using different methods. Employing theories of the perturbations on ranges of nonlinear operators to discuss the existence of solutions of NBVPs is one of the important methods, related work can be found in [1–7]. In 2010, Wei, Agarwal and Wong [8] studied the following nonlinear parabolic boundary value problem involving the generalized p-Laplacian

$$\begin{cases} \dfrac{\partial u}{\partial t} - \operatorname{div}[(C(x,t)+|\nabla u|^2)^{\frac{p-2}{2}}\nabla u] + \varepsilon|u|^{p-2}u = f(x,t), \quad (x,t)\in\Omega\times(0,T) \\ -\langle \vartheta, (C(x,t)+|\nabla u|^2)^{\frac{p-2}{2}}\nabla u\rangle \in \beta(u) - h(x,t), \quad (x,t)\in\Gamma\times(0,T) \\ = u(x,0) = u(x,T) \quad \text{a.e. } x\in\Omega, \end{cases} \tag{1.1}$$

where $0 \le C(x,t) \in L^p(\Omega\times(0,T))$, ε is a non-negative constant and ϑ denotes the exterior normal derivative of Γ. Based on the properties of the ranges for pseudo-monotone operators and maximal monotone operators presented in [9], it is shown that (1.1) has solutions $L^p(0,T;W^{1,p}(\Omega))$, where $2\le p<+\infty$.

Recently, Wei, Agarwal and Wong [10] studied the following elliptic p-Laplacian-like equation with mixed boundary conditions

$$\begin{cases} \operatorname{div}[(C(x)+|\nabla u|^2)^{\frac{s}{2}}|\nabla u|^{m-1}\nabla u] + \varepsilon|u|^{q-2}u + g(x,u(x)) = f(x) & \text{in } \Omega, \\ -\langle \vartheta, (C(x)+|\nabla u|^2)^{\frac{s}{2}}|\nabla u|^{m-1}\nabla u\rangle \in \beta_x(u) & \text{on } \Gamma. \end{cases} \tag{1.2}$$

By using the perturbation results on the ranges for m-accretive mappings presented in [1], it is shown that (1.2) has solutions in $L^p(\Omega)$ under some conditions, where $\frac{2N}{N+1}<p<+\infty$, $1\le q<+\infty$ if $p\ge N$, and $1\le q\le \frac{Np}{N-p}$ if $p<N$ for $N\ge 1$.

In 2012, Wei, Agarwal and Wong [11] studied the following integro-differential equation with generalized p-Laplacian operator

$$\begin{cases} \dfrac{\partial u}{\partial t} - \operatorname{div}[(C(x,t)+|\nabla u|^2)^{\frac{p-2}{2}}\nabla u] + \varepsilon|u|^{q-2}u + a\dfrac{\partial}{\partial t}\displaystyle\int_\Omega u(x,t)\mathrm{d}x = f(x,t), \\ \hfill (x,t)\in\Omega\times(0,T), \\ -\langle \vartheta, (C(x,t)+|\nabla u|^2)^{\frac{p-2}{2}}\nabla u\rangle \in \beta_x(u(x,t)), \quad (x,t)\in\Gamma\times(0,T), \\ u(x,0) = u(x,T), \quad x\in\Omega. \end{cases} \tag{1.3}$$

By using some results on the ranges for bounded pseudo-monotone operator and maximal monotone operator presented in [3, 12, 13], they obtain that (1.3) has solutions in $L^p(0,T;W^{1,p}(\Omega))$ for $1<q\le p<+\infty$.

In this paper, motivated by the previous work, we shall consider the following parabolic p-Laplacian-like problem

$$\begin{cases} \alpha(\dfrac{\partial u}{\partial t}) - \operatorname{div}[(C(x,t)+|\nabla u|^2)^{\frac{s}{2}}|\nabla u|^{m-1}\nabla u] + \varepsilon|u|^{q-2}u + g(x,u,\nabla u) = f(x,t), \\ \hfill (x,t)\in\Omega\times(0,T), \\ -\langle \vartheta, (C(x,t)+|\nabla u|^2)^{\frac{s}{2}}|\nabla u|^{m-1}\nabla u\rangle \in \beta_x(u(x,t)), \quad (x,t)\in\Gamma\times(0,T), \\ u(x,0) = u(x,T), \quad x\in\Omega. \end{cases} \tag{1.4}$$

In (1.4), α is the subdifferential of j, i.e., $\alpha=\partial j$, where $j: R\to R$ is a proper, convex and lower-semi continuous function, β_x is the subdifferential of φ_x, i.e., $\beta_x \equiv \partial\varphi_x$, where