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SHARP ESTIMATES OF ALL HOMOGENEOUS EXPANSIONS FOR A SUBCLASS OF QUASI-CONVEX MAPPINGS OF TYPE $\mathbb{B}$ AND ORDER α IN SEVERAL COMPLEX VARIABLES*

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Abstract In this article, first, the sharp estimates of all homogeneous expansions for a subclass of quasi-convex mappings of type $\mathbb{B}$ and order α on the unit ball in complex Banach spaces are given. Second, the sharp estimates of all homogeneous expansions for the above generalized mappings on the unit polydisk in $\mathbb{C}^n$ are also established. In particular, the sharp estimates of all homogeneous expansions for a subclass of quasi-convex mappings (include quasi-convex mappings of type $\mathbb{A}$ and quasi-convex mappings of type $\mathbb{B}$) in several complex variables are get accordingly. Our results state that a weak version of the Bieberbach conjecture for quasi-convex mappings of type $\mathbb{B}$ and order α in several complex variables is proved, and the derived conclusions are the generalization of the classical results in one complex variable.

Key words homogeneous expansion; quasi-convex mapping of type $\mathbb{B}$ and order α; quasi-convex mapping; quasi-convex mapping of type $\mathbb{A}$; quasi-convex mapping of type $\mathbb{B}$

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1 Introduction

In geometric function theorey of one complex variable, people show great interest in the following classical theorem.

Theorem A (see [12]) If $f(z) = z + \sum\limits_{n=2}^{\infty} a_n$ is a normalized biholomorphic convex function of order α on the unit disk U in $\mathbb{C}$, then

$$|a_n| \le \frac{1}{n!}\prod_{k=2}^{n}(k-2\alpha), \quad n = 2, 3, \cdots,$$

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and the above estimates are sharp.

We are naturally to ask whether the corresponding result in several complex variables holds or not? In this article, we shall in part provide an affirmative answer.

Concerning the sharp estimates of all homogeneous expansions for a subclass of quasi-convex mappings (include quasi-convex mappings of type $\mathbb{A}$ and quasi-convex mappings of type $\mathbb{B}$) in several complex variables, it was shown that the above result in general is invalid (see [13]). However, on a special domain, such as the unit polydisk in $\mathbb{C}^n$, Liu [7], Liu and Liu [9] obtained the sharp estimates of all homogeneous expansions for quasi-convex mappings (include quasi-convex mappings of type $\mathbb{A}$ and quasi-convex mappings of type $\mathbb{B}$) under different restricted conditions respectively. On the other hand, Liu and Liu [8] derived the sharp estimates of all homogeneous expansions for a subclass of quasi-convex mappings of type $\mathbb{B}$ and order α (include quasi-convex mappings, quasi-convex mappings of type $\mathbb{A}$ and quasi-convex mappings of type $\mathbb{B}$). We mention that the family of quasi-convex mappings of type $\mathbb{B}$ and order α is a significant family of holomorphic mappings in several complex variables, and the Bieberbach conjecture in several complex variables (i.e., the sharp estimates of all homogeneous expansions for biholomorphic starlike mappings on the unit polydisk in $\mathbb{C}^n$ hold) (see [1, 3, 10]) is a very significant and extremal difficult problem. Owing to this reason, the sharp estimates of all homogeneous expansions for quasi-convex mappings of type $\mathbb{B}$ and order α seem to be a meaningful problem as well.

Let X denote a complex Banach space with the norm $\|.\|$, let X^* denote the dual space of X, let B be the open unit ball in X, and let U be the Euclidean open unit disk in $\mathbb{C}$. We also denote by U^n the open unit polydisk in $\mathbb{C}^n$, B^n the Euclidean unit ball in $\mathbb{C}^n$ and $\mathbb{N}^*$ the set of all positive integers. Let ∂U^n denote the boundary of U^n, $(\partial U)^n$ be the distinguished boundary of U^n. Let the symbol $'$ mean transpose. For each $x \in X\backslash\{0\}$, we define

$$T(x) = \{T_x \in X^* : \ \|T_x\| = 1, T_x(x) = \|x\|\}.$$

By the Hahn-Banach theorem, $T(x)$ is nonempty.

Let $H(B)$ be the set of all holomorphic mappings from B into X. We know that if $f \in H(B)$, then

$$f(y) = \sum_{n=0}^{\infty} \frac{1}{n!} D^n f(x)((y-x)^n)$$

for all y in some neighborhood of $x \in B$, where $D^n f(x)$ is the nth-Fréchet derivative of f at x, and for $n \geq 1$,

$$D^n f(x)((y-x)^n) = D^n f(x)(\underbrace{y-x, \cdots, y-x}_{n}).$$

Furthermore, $D^n f(x)$ is a bounded symmetric n-linear mapping from $\prod_{j=1}^{n} X$ into X.

We say that a holomorphic mapping $f : B \to X$ is biholomorphic if the inverse f^{-1} exists and is holomorphic on the open set $f(B)$. A mapping $f \in H(B)$ is said to be locally biholomorphic if the Fréchet derivative $Df(x)$ has a bounded inverse for each $x \in B$. If $f : B \to X$ is a holomorphic mapping, then we say that f is normalized if $f(0) = 0$ and $Df(0) = I$, where I represents the identity operator from X into X.

We say that a normalized biholomorphic mapping $f : B \to X$ is a starlike mapping if $f(B)$ is a starlike domain with respect to the origin.