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NEW LOWER BOUNDS FOR LEE DISCREPANCY ON TWO AND THREE MIXED LEVELS FACTORIALS*

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Abstract The objective of this paper is to study the issue of uniformity on asymmetrical designs with two and three mixed levels in terms of Lee discrepancy. Based on the known formulation, we present a new lower bound of Lee discrepancy of fractional factorial designs with two and three mixed levels. Our new lower bound is sharper and more valid than other existing lower bounds in literature, which is a useful complement to the lower bound theory of discrepancies.

Key words U-type design; Lee discrepancy; uniform design; lower bound

2010 MR Subject Classification 62K15; 62K05

1 Introduction

Uniform designs are important methods of computer experiments and robust experimental designs, which spread experimental points uniformly on the experimental domain. The measure of uniformity plays a key role in the construction of uniform designs. Many discrepancies in quasi-Monte Carlo methods were employed to measure the uniformity of designs. For details, we can refer to [4]. A design with the smallest discrepancy value is called a uniform design.

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Among these discrepancies, the Lee discrepancy proposed by [5] was regarded more reasonable and practicable. In this paper, we use Lee discrepancy as the uniformity measure.

Recently, there was considerable interest in exploring the issue of lower bounds for Lee discrepancy, because lower bounds of Lee discrepancy can be used as a benchmark not only in searching for uniform designs but also in helping to validate that some good designs are in fact uniform or not. Many authors tried to find lower bounds to Lee discrepancy so as to obtain most efficient designs. Obviously, a design whose Lee discrepancy value achieves a strict lower bound is a uniform design with respect to this discrepancy. [5] gave some lower bounds for both symmetric and asymmetric U-type designs. [6] provided two improved lower bounds of Lee discrepancy of fractional factorials with two or three levels. [2] obtained a lower bound of Lee discrepancy of asymmetrical factorials with two- and three-levels. [7] studied the uniformity in terms of Lee discrepancy of double designs. Note that two and three mixed levels factorials are most popular among mixed factorials. The present paper aims at obtaining some improved lower bounds for the Lee discrepancy on factorials with two and three mixed levels.

The remainder of this paper is organized as follows: Section 2 provides the notations and preliminaries. In Section 3, lower bounds for Lee discrepancy are provided. Some illustrative examples are provided in Section 4.

2 Notations and Preliminaries

An m-level U-type design belonging to a class $\mathcal{U}(n;m^s)$ of designs is an $n\times s$ array with entries from the set $\{0,1,\cdots,m-1\}$ such that in each column each of entries of the set $\{0,1,\cdots,m-1\}$ appears equally often. A U-type design with s_1 two-level factors and s_2 three-level factors belonging to a class $\mathcal{U}(n;2^{s_1}3^{s_2})$ of designs corresponds to an $n\times s$ $(s=s_1+s_2)$ array $X=(x_1,\cdots,x_s)$ such that each of entries in each column x_i $(1\le i\le s_1)$ takes values from a set of $\{0,1\}$ equally often, and each of entries in x_i $(s_1+1\le i\le s)$ takes values from a set of $\{0,1,2\}$ equally often.

For any design $d\in\mathcal{U}(n;2^{s_1}3^{s_2})$, according to [5], Lee-discrepancy value of d, denoted by $LD(d)$, can be calculated by the following formula

$$[LD(d)]^2=-\left(\frac{3}{4}\right)^{s_1}\left(\frac{7}{9}\right)^{s_2}+\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\prod_{k=1}^{s_1}(1-\alpha_{ij}^k)\right]\left[\prod_{k=s_1+1}^{s}(1-\beta_{ij}^k)\right], \qquad (2.1)$$

where for $i,j=1,\cdots,n$,

$$\left.\begin{aligned}\alpha_{ij}^k&=\min\left\{\frac{|x_{ik}-x_{jk}|}{2},1-\frac{|x_{ik}-x_{jk}|}{2}\right\},\ k=1,2,\cdots,s_1;\\ \beta_{ij}^k&=\min\left\{\frac{|x_{ik}-x_{jk}|}{3},1-\frac{|x_{ik}-x_{jk}|}{3}\right\},\ k=s_1+1,\cdots,s.\end{aligned}\right\} \qquad (2.2)$$

For any design $d\in\mathcal{U}(n;m^s)$ and its corresponding $n\times s$ matrix $(x_1,x_2,\cdots,x_s)$, where $x_j=(x_{1j},x_{2j},\cdots,x_{nj})^T$, we define

$$\delta_{l_1l_2}(d)=\sum_{j=1}^{s}\delta_{l_1l_2}(x_j),$$

where $\delta_{l_1l_2}(x_j)$ is the Kronecker delta function, equal to 1 if $x_{l_1j}=x_{l_2j}$ and 0 otherwise. Then we call $\delta_{l_1l_2}(d)$ as the coincidence number of the l_1th and l_2th rows for d.