Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2016,**36B**(4):973–1014
Mathematica Acta Scientia
数学物理学报
http://actams.wipm.ac.cn

GLOBAL EXISTENCE OF WEAK SOLUTIONS TO THE NON-ISOTHERMAL NEMATIC LIQUID CRYSTALS IN 2D*

Jinkai LI (李进开)
The Institute of Mathematical Sciences, The Chinese University of Hong Kong, Hong Kong, China
Department of Computer Science and Applied Mathematics, Weizmann Institute of Science Rehovot 76100, Israel
E-mail: *jklimath@gmail.com*

Zhouping XIN (辛周平)†
The Institute of Mathematical Sciences, The Chinese University of Hong Kong, Hong Kong, China
E-mail: *zpxin@ims.cuhk.edu.hk*

Dedicated to Professor Boling Guo on the occasion of his 80th birthday

Abstract In this article, we prove the global existence of weak solutions to the non-isothermal nematic liquid crystal system on $\mathbb{T}^2$, on the basis of a new approximate system which is different from the classical Ginzburg-Landau approximation. Local in space energy inequalities are employed to recover the estimates on the second order spatial derivatives of the director fields locally in time, which cannot be derived from the basic energy balance. It is shown that these weak solutions satisfy the temperature equation, and also the total energy equation but away from at most finite many "singular" times, at which the energy concentration occurs and the director field losses its second order derivatives.

Key words Global weak solutions; non-isothermal; nematic liquid crystals

2010 MR Subject Classification 35D30; 76A15

1 Introduction

Liquid crystal is an intermediate state between liquids and solids. There are three phases of liquid crystals: nematic, cholesteric, and smectic, and the smectic phase is divided into several subtypes (smectic A, smectic C and etc.). In the nematic phase the long axes of the constituent molecules tend to align parallel to each other along some common preferred direction. The first widely accepted dynamic theory for nematic liquid crystals was formulated by Ericksen [1] in 1961, which was later completed in 1968 by Leslie [2], who derived suitable constitutive

*Received February 26, 2016. This research is supported partially by Zheng Ge Ru Foundation, Hong Kong RGC Earmarked Research Grants 14305315, CUHK4041/11P and CUHK4048/13P, a Focus Area Grant from The Chinese University of Hong Kong, a Croucher Foundation-CAS Joint Grant, and a NSFC/RGC Joint Research Scheme (N-CUHK443/14).

†Corresponding author

equations. Their theory, now referred to as the Ericksen-Leslie dynamic theory, is one of the most successful theories used to model many dynamic phenomena in nematic liquid crystals.

As shown by Leslie [3], the mechanical balance equations must be supplemented by an equation for the thermal field θ. The most simple non-isothermal model is that for the so called elastically isotropic liquid crystals, that is, the elastic coefficients (the Frank's constants) have the same values. This is equivalently to say that the Oseen-Frank free energy density has the simple expression $\frac{\lambda}{2}|\nabla d|^2$, for a positive number λ. In this case, it leads us to consider the following system (see Appendix for the derivation)

$$\partial_t u + (u\cdot\nabla)u + \nabla p = \mathrm{div}(S + \sigma^{\mathrm{nd}}), \tag{1.1}$$

$$\mathrm{div} u = 0, \qquad |d| = 1, \tag{1.2}$$

$$\partial_t d + (u\cdot\nabla)d - (d\cdot\nabla)u + (d\cdot\nabla u\cdot d)d = \gamma(\Delta d + |\nabla d|^2 d), \tag{1.3}$$

$$\partial_t E + \mathrm{div}\,(uE) + \mathrm{div}(q + up) = \mathrm{div}\left(S\cdot u + \sigma^{\mathrm{nd}}\cdot u + \lambda(\gamma\Delta d + (d\cdot\nabla)u)\cdot\nabla d)\right), \tag{1.4}$$

where the velocity u, orientation field d, pressure p, and total energy E are the unknowns, and λ and γ are two positive constants.

The total energy E has the expression

$$E = \frac{|u|^2}{2} + \frac{\lambda}{2}|\nabla d|^2 + \theta,$$

where θ is the absolute temperature. The energy flux q in equation (1.4) is given by

$$q = -\kappa(\theta)\nabla\theta - h(\theta)(d\cdot\nabla\theta)d, \tag{1.5}$$

with two positive functions κ and h depending on the temperature θ. Note that (1.5) is an extension of Fourier's law for liquid crystals. The notations S and σ^{nd} denote the dissipative and nondissipative part of the stress, respectively, given by

$$S = \mu(\theta)(\nabla u + \nabla u^T) + \frac{\lambda}{\gamma}(d\cdot\nabla u\cdot d)d\otimes d, \tag{1.6}$$

and

$$\sigma^{\mathrm{nd}} = -\lambda\nabla d\odot\nabla d - \lambda(\Delta d + |\nabla d|^2 d)\otimes d, \tag{1.7}$$

where ∇u^T is the transform of ∇u and $\nabla d\odot\nabla d$ is a $N\times N$ matrix whose (i,j)-th entry is $\partial_i d\cdot\partial_j d$, $1\le i,j\le N$.

System (1.1)–(1.4) is a simplified version of the general non-isothermal Ericksen-Lselie system, by assuming that the material is elastically isotropic and specifying some Leslie coefficients (see the appendix for details). The general non-isothermal Ericksen-Leslie system can be found, for example, in the books by Oswald-Pieranski [4] (Chapter B III2) and Sonnet-Virga [5] (Chapter 3). Besides, as explained in the appendix of this article, (1.1)–(1.4) is the limiting system, as $\varepsilon\to 0$, of that introduced by Feireisl-Fremond-Rocca-Schimperna in [6], with the penalty term $f(d)$ there taken as the Ginzburg-Landau type, that is $f(d) = \frac{1-|d|^2}{\varepsilon^2}d$, in other words, the system considered in [6] can be viewed as the Ginzburg-Landau type approximation of system (1.1)–(1.4). The relevant simplified isothermal model was proposed by Lin in [7] and later analyzed by Lin-Liu in [8], where the penalty function $f(d)$ is considered instead of $|\nabla d|^2 d$. The model proposed in [7, 8] is a considerably simplified version of the famous Ericksen-Leslie model introduced by Ericksen [1] and Leslie [2] in the 1960's. Global existence of weak solutions to the

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