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LARGE TIME ASYMPTOTIC BEHAVIOR OF THE COMPRESSIBLE NAVIER-STOKES EQUATIONS IN PARTIAL SPACE-PERIODIC DOMAINS*

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Dedicated to Professor Boling Guo on the occasion of his 80th birthday

Abstract In this article, we study the large time behavior of the 3-D isentropic compressible Navier-Stokes equation in the partial space-periodic domains, and simultaneously show that the related profile systems can be described by like Navier-Stokes equations with suitable "pressure" functions in lower dimensions. Our proofs are based on the energy methods together with some delicate analysis on the corresponding linearized problems.

Key words Large time behavior; profile system; energy method; partial space-periodic domain; Fourier series

2010 MR Subject Classification 35Q30; 76N10

1 Introduction and Main Results

In this article, we consider the 3-D isentropic compressible Navier-Stokes equation for $(t, z) \in [0, +\infty) \times \Omega$:

$$\begin{cases} \partial_t \rho + \mathrm{div}\ m = 0, \\ \partial_t m + \mathrm{div}\ \Big(\dfrac{m \otimes m}{\rho}\Big) + \nabla p(\rho) = \mu \Delta\Big(\dfrac{m}{\rho}\Big) + (\mu + \mu')\nabla \mathrm{div}\ \Big(\dfrac{m}{\rho}\Big), \end{cases} \tag{1.1}$$

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where $\Omega = \mathbb{T}^\ell \times \mathbb{R}^{3-\ell}$, $\mathbb{T}^\ell = [0, 2\pi]^\ell$ is the ℓ-dimensional torus ($1 \le \ell \le 3$), $z = (z_1, z_2, z_3)$, $\rho = \rho(t, z)$ is the density, $m = (m^1, m^2, m^3)(t, z)$ is the momentum, $p = p(\rho)$ is the pressure with $P'(\rho) > 0$ and $P''(\rho) > 0$ for $\rho > 0$, and μ and μ' are the first and the second viscosity coefficient respectively, which satisfy $\mu > 0$ and $\frac{2}{3}\mu + \mu' \ge 0$. The initial data of (1.1) are given as follows

$$(\rho, m)(0, z) = (1 + \rho_0(z), m_0(z)), \tag{1.2}$$

where $(\rho_0, m_0) \in H^4(\Omega) \times (H^4(\Omega))^3$, and $1 + \rho_0(z) > 0$ for $z \in \Omega$.

It is obvious that $(\rho, m) = (1, 0)$ is a steady solution of (1.1) with the initial data $(\rho, m)(0, z) = (1, 0)$. We will be concerned with the perturbation problem of (1.1) to this constant state. Denote

$$\nu_1 = \mu,\ \nu_2 = \mu + \mu',\ \gamma = \sqrt{p'(1)}.$$

As in [14, 15], if set $\phi = \gamma(\rho - 1)$, then (1.1)–(1.2) can be rewritten as

$$\begin{cases} \partial_t \phi + \gamma \text{div } m = 0, \\ \partial_t m - \nu_1 \Delta m - \nu_2 \nabla \text{div } m + \gamma \nabla \phi = G(\phi, m), \\ (\phi, m)(0, z) = (\phi_0, m_0)(z), \end{cases} \tag{1.3}$$

where $\phi_0(z) = \gamma \rho_0(z)$ and

$$\begin{aligned} G(\phi, m) = &- \text{div } \left(\frac{\gamma}{\phi + \gamma} m \otimes m \right) - \nu_1 \Delta \left(\frac{\phi}{\phi + \gamma} m \right) - \nu_2 \nabla \text{div } \left(\frac{\phi}{\phi + \gamma} m \right) \\ &- \nabla \left\{ \frac{\phi^2}{\gamma^2} \int_0^1 (1 - \theta)^2 p''\big(1 + \frac{\theta \phi}{\gamma}\big) \mathrm{d}\theta \right\}. \end{aligned}$$

We now introduce some notations for later uses. The Fourier transformation of the function $f \in L^1(\mathbb{T}^\ell \times \mathbb{R}^{3-\ell})$ is denoted by

$$\mathcal{F}(f)(k, \xi) = \hat{f}(k, \xi) = \int_{\mathbb{T}^\ell} \int_{\mathbb{R}^{3-\ell}} e^{-\mathrm{i}(k \cdot x + \xi \cdot y)} f(x, y) \mathrm{d}x \mathrm{d}y,$$

where $z = (x, y)$, $x = (z_1, \cdots, z_l) \in \mathbb{T}^\ell$, $y = (y_1, \cdots, y_{3-\ell}) = (z_{\ell+1}, \cdots, z_3) \in \mathbb{R}^{3-\ell}$, $k = (k_1, \cdots, k_\ell) \in \mathbb{Z}^\ell$, $\xi = (\xi_1, \cdots, \xi_{3-\ell}) \in \mathbb{R}^{3-\ell}$. The inverse Fourier transformation of the sequence $\{g(k, \xi)\}_{k \in \mathbb{Z}^l}$ is defined as

$$(\mathcal{F}^{-1} g)(z) = \frac{1}{(2\pi)^\ell} \sum_{k \in \mathbb{Z}^\ell} e^{\mathrm{i} k \cdot x} \int_{\mathbb{R}^{3-\ell}} e^{\mathrm{i} \xi \cdot y} g(k, \xi) \mathrm{d}\xi.$$

Write the mean value of $f(z)$ over $\mathbb{T}^\ell$ as $\bar{f}(y)$:

$$\bar{f}(y) = \frac{1}{(2\pi)^\ell} \int_{\mathbb{T}^\ell} f(x, y) \mathrm{d}x.$$

In addition, we define

$$\text{div }' v = \partial_{y_1} v_1 + \cdots + \partial_{y_{3-\ell}} v_{3-\ell},\ \nabla' = (\partial_{y_1}, \cdots, \partial_{y_{3-\ell}})^T,\ \Delta' = \partial_{y_1}^2 + \cdots + \partial_{y_{3-\ell}}^2,$$

where $v = (v_1, \cdots, v_{3-\ell})^T$. And we denote $\|u(t, \cdot)\|_{L^p(\Omega)}$ as $\|u\|_p$ for $1 \le p \le \infty$.

For different ℓ, our main results in this article are

Theorem 1.1 For $\ell = 1$, if $u_0 \in H^4(\mathbb{T} \times \mathbb{R}^2)$ and $\|u_0\|_{H^4 \cap L^1} \le \varepsilon$, then for small $\varepsilon > 0$, (1.3) has a global solution $u(t, z) = (\phi, m)(t, z) \in C([0, +\infty), H^4(\mathbb{T} \times \mathbb{R}^2)) \cap C^1([0, +\infty), H^2(\mathbb{T} \times \mathbb{R}^2))$ satisfying for $t \to +\infty$

$$\|\partial_z^k u\|_2 = O(t^{-\frac{1}{2} - \frac{k}{2}}),\quad k = 0, 1, \tag{1.4}$$

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