Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2016,**36B**(4):1192–1214
Acta Mathematica Scientia 数学物理学报
http://actams.wipm.ac.cn

GLOBAL STABILITY OF WAVE PATTERNS FOR COMPRESSIBLE NAVIER-STOKES SYSTEM WITH FREE BOUNDARY*

Xiaohong QIN (秦晓红)
Department of Mathematics, Nanjing University of Science and Technology, Nanjing 210094, China
E-mail: *xqin@amss.ac.cn*

Teng WANG (王腾)
Department of Mathematics, School of Science, Beijing Forestry University, Beijing 100083, China
E-mail: *tengwang@amss.ac.cn*

Yi WANG (王益)†
Institute of Applied Mathematics, AMSS, CAS, Beijing 100190, China
E-mail: *wangyi@amss.ac.cn*

Dedicated to Professor Boling Guo on the occasion of his 80th birthday

Abstract In this article, we investigate the global stability of the wave patterns with the superposition of viscous contact wave and rarefaction wave for the one-dimensional compressible Navier-Stokes equations with a free boundary. It is shown that for the ideal polytropic gas, the superposition of the viscous contact wave with rarefaction wave is nonlinearly stable for the free boundary problem under the large initial perturbations for any $\gamma > 1$ with γ being the adiabatic exponent provided that the wave strength is suitably small.

Key words Compressible Navier-Stokes system; free boundary; combination of viscous contact and rarefaction wave; nonlinear stability

2010 MR Subject Classification 76N10

1 Introduction

The one-dimensional compressible Navier-Stokes equations in the Eulerian coordinate read

$$\begin{cases} \tilde{\rho}_t + (\tilde{\rho}\tilde{u})_{\tilde{x}} = 0, \\ (\tilde{\rho}\tilde{u})_t + (\tilde{\rho}\tilde{u}^2 + \tilde{p})_{\tilde{x}} = \mu\tilde{u}_{\tilde{x}\tilde{x}}, \\ \left[\tilde{\rho}\left(\tilde{e} + \dfrac{\tilde{u}^2}{2}\right)\right]_t + \left[\tilde{\rho}\left(\tilde{e} + \dfrac{\tilde{u}^2}{2}\right) + \tilde{p}\tilde{u}\right]_{\tilde{x}} = \kappa\tilde{\theta}_{\tilde{x}\tilde{x}} + \mu(\tilde{u}\tilde{u}_{\tilde{x}})_{\tilde{x}}, \end{cases} \tag{1.1}$$

*Received November 4, 2015; revised January 29, 2016. The research of X H Qin was supported by NSFC Grant No. 11171153, the research of T Wang was supported by the Fundamental Research Funds for the Central Universities No. 2015ZCQ-LY-01 and No. BLX2015-27, and the research of Y Wang was supported by NSFC Grant No. 11322106.

†Corresponding author

where $\tilde{u}(\tilde{x},t)$ is the velocity, $\tilde{\rho}(\tilde{x},t)>0$ is the density, $\tilde{\theta}(\tilde{x},t)>0$ is the absolute temperature, $\tilde{p}=\tilde{p}(\tilde{\rho},\tilde{\theta})$ is the pressure, and $\tilde{e}=\tilde{e}(\tilde{\rho},\tilde{\theta})$ is the internal energy of the gas in gas dynamics, while μ and κ denote the viscosity and the heat-conductivity of the gas, respectively. Here, we study the ideal polytropic gas, that is,

$$\tilde{p}=R\tilde{\rho}\tilde{\theta}=A\tilde{\rho}^{\gamma}e^{\frac{\gamma-1}{R}\tilde{s}},\quad \tilde{e}=c_{\nu}\tilde{\theta},$$

where $\tilde{s}=\tilde{s}(\tilde{\rho},\tilde{\theta})$ is the entropy, $\gamma>1$ is the adiabatic exponent, $c_{\nu}=\frac{R}{\gamma-1}$ is the specific heat, and both A and R are positive constants.

We consider the system (1.1) in the region $\tilde{x}>\tilde{x}(t)$, with the free boundary $\tilde{x}=\tilde{x}(t)$ defined by

$$\begin{cases}\dfrac{\mathrm{d}\tilde{x}(t)}{\mathrm{d}t}=\tilde{u}(\tilde{x}(t),t), & t>0,\\ \tilde{x}(0)=0,\end{cases}\tag{1.2}$$

and the free boundary conditions

$$(\tilde{p}-\mu\tilde{u}_{\tilde{x}})\big|_{\tilde{x}=\tilde{x}(t)}=p_0,\qquad \tilde{\theta}\big|_{\tilde{x}=\tilde{x}(t)}=\theta_->0,\tag{1.3}$$

which means that the gas is attached at the boundary $\tilde{x}=\tilde{x}(t)$ with the fixed outer pressure $p_0>0$ and the prescribed temperature $\theta_->0$. The initial data is given by

$$(\tilde{\rho},\tilde{u},\tilde{\theta})\big|_{t=0}=\left(\tilde{\rho}_0,\tilde{u}_0,\tilde{\theta}_0\right)(\tilde{x})\to(\rho_+,u_+,\theta_+)\quad\text{as}\ \ \tilde{x}\to+\infty,\tag{1.4}$$

where $\rho_+>0$, $\theta_+>0$, and u_+ are prescribed constants and we assume $\tilde{\theta}_0\big|_{\tilde{x}=\tilde{x}(t)}=\theta_-$ as the compatibility condition.

As it is convenient to use the Lagrangian coordinate in spatial one-dimensional case, we transform the Eulerian coordinates $(\tilde{x},t)$ to the Lagrangian coordinates (x,t) by

$$x=\int_{\tilde{x}(t)}^{\tilde{x}}\tilde{\rho}(y,t)\mathrm{d}y,\quad t=t,$$

and then the free boundary value problem (1.1)–(1.4) is changed into the half space problem

$$\begin{cases}v_t-u_x=0,\\ u_t+p_x=\mu\left(\dfrac{u_x}{v}\right)_x, & x>0,\ t>0,\\ \left(e+\dfrac{u^2}{2}\right)_t+(pu)_x=\left(\kappa\dfrac{\theta_x}{v}+\mu\dfrac{uu_x}{v}\right)_x,\end{cases}\tag{1.5}$$

with the initial and boundary conditions

$$\begin{cases}\theta|_{x=0}=\theta_-,\\ \left(p(v,\theta)-\mu\dfrac{u_x}{v}\right)(0,t)=p_0,\quad t>0,\\ (v,u,\theta)(x,0)=(v_0,u_0,\theta_0)(x)\to(v_+,u_+,\theta_+)\quad\text{as}\ x\to+\infty,\end{cases}\tag{1.6}$$

where $u(x,t)=\tilde{u}(\tilde{x},t)$, $\theta(x,t)=\tilde{\theta}(\tilde{x},t)$, and $v_+=\rho_+^{-1}$; and $v=v(x,t)=\tilde{\rho}^{-1}(\tilde{x},t)$ denotes the specific volume.