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PERSISTENCE AND THE GLOBAL DYNAMICS OF THE POSITIVE SOLUTIONS FOR A RATIO-DEPENDENT PREDATOR-PREY SYSTEM WITH A CROWDING TERM IN THE PREY EQUATION*

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Abstract This paper deals with the global dynamical behaviors of the positive solutions for a parabolic type ratio-dependent predator-prey system with a crowding term in the prey equation, where it is assumed that the coefficient of the functional response is less than the coefficient of the intrinsic growth rates of the prey species. We demonstrated some special dynamical behaviors of the positive solutions of this system which the persistence of the coexistence of two species can be obtained when the crowding region in the prey equation only is designed suitably. Furthermore, we can obtain that under some conditions, the unique positive steady state solution of the system is globally asymptotically stable.

Key words ratio-dependent predator-prey system; crowding effect; persistence; global stability

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1 Introduction

we will investigate the persistence and the dynamical behaviors of the positive solutions for the following predator-prey system

$$\begin{cases} u_t - d_1\triangle u = \lambda u - a(x)u^2 - \dfrac{buv}{u+mv}, & x\in\Omega, \quad t>0, \\ v_t - d_2\triangle v = \mu v - v^2 + \dfrac{cuv}{u+mv}, & x\in\Omega, \quad t>0, \\ \partial_\nu u = \partial_\nu v = 0, & x\in\partial\Omega, \quad t>0, \\ u(x,0)=u_0(x), \quad v(x,0)=v_0(x), & x\in\overline{\Omega}, \end{cases} \tag{1.1}$$

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where Ω is a bounded domain in $\mathbf{R}^N$ with smooth boundary $\partial\Omega$, $N \geq 1$, ν is the outward unit normal on $\partial\Omega$, and $\partial_\nu := \partial/\partial\nu$; $\lambda, \mu, b, c, m, d_1$ and d_2 are constants, and all of these constants are positive except μ which may take negative values; $a(x)$ is a nonnegative continuous function in $\overline{\Omega}$. Moreover, there exists a subregion $\Omega_0 \subset \Omega$, such that $a(x) \equiv 0$ in $\overline{\Omega}_0$ and $a(x) > 0$ in $\overline{\Omega} \setminus \overline{\Omega}_0$. We assume that Ω_0 is an open and connected subset of Ω, and $\partial\Omega_0 \in C^2$. The homogeneous Neumann boundary conditions indicate that system (1.1) is self-contained with zero population flux across the boundary. For the meanings of other terms and coefficients in (1.1), one can refer to the references [41–43, 50, 51].

System (1.1) arises in mathematical biology as a ratio-dependent predator-prey model of two species which are interacting each other and migrating in the same habitat Ω. To our knowledge, the classical prey-dependent predator-prey models exhibit the 'paradox of enrichment' and the so-called 'biological control paradox' [1, 2, 20]; but the ratio-dependent predator-prey model produces neither a paradox of enrichment nor the biological control paradox [25–27]. Therefore, the ratio-dependent predator-prey model should be a more reasonable model in the prey-dependent predator-prey models.

In addition, the coefficient $a(x)$ in system (1.1) is not constant, but a function of the space variable x. As far as we know, in the spatial population models, on account of the effect of the environment, some coefficients such as the growth rates, the crowding effects and the population interaction rates, are usually replaced by functions of the space variable x. The spatially heterogeneous models are very meaningful and valuable in the control to the alien species or the protection zone etc [11–17, 28–31, 37, 47, 48]. Moreover, many pioneers such as Brézis and Oswald [5], Cantrell and Cosner [6], Cirstea and Radulescu [7–9], Du and his coauthors [11–19], Fraile and his coauthors [21], García-Melián and his coauthors [23, 24], López-Gómez and his coauthors [3, 23, 28–36] and Ouyang [40] etc, had many outstanding fundamental works in this field. For the class of population models in a spatially heterogeneous environment, it has been observed that in general, the behaviors of the solutions of the class of population models are very sensitive to the change of certain coefficient functions in part of the underlying spatial region, and the coexistence of two species depends strictly on the relationship of the growth rates λ with the principal eigenvalue $\lambda_1^D(\Omega_0)$. We find that these results of [3, 11–17, 19, 28–31, 37, 47, 48] were different from those ones of the corresponding systems in the spatially homogeneous environment [22, 38, 39, 41–43, 45, 46, 49–51]. One can see [3, 5–9, 11–19, 21, 23, 24, 28–37, 40, 47, 48] and references therein for a more detailed discussion on the models in a spatially heterogeneous environment.

In this paper, we will investigate the impact of the subregion Ω_0 on the persistence and the dynamical behaviors of the positive solutions of system (1.1) with $\lambda > b$. System (1.1) is singular and non-differential at the point $(u, v) = (0, 0)$, but we may re-define $\frac{buv}{u+mv} := 0$ at $(0, 0)$. To keep things simple, we suppose that $d_1 = d_2 = m = 1$. In [52], we obtained the existence, uniqueness and stability of the positive steady state solution of (1.1) provided $b < \lambda < \lambda_1^D(\Omega_0)$ and $\mu > -c$, and the nonexistence of the positive steady state solution of (1.1) when $\mu \leq -c$, or $\lambda > \lambda_1^D(\Omega_0)$ and $\mu > -c$. In this paper, by using the comparison principle, we will obtain that the positive solutions of (1.1) is persist only when $b < \lambda < \lambda_1^D(\Omega_0)$ and $\mu > -c$; Furthermore, by using the energy functional methods, we will demonstrate that the unique positive steady state solution of (1.1) is globally asymptotically stable when μ is large

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