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STRONG CONVERGENCE THEOREMS FOR EQUILIBRIUM PROBLEM AND BREGMAN TOTALLY QUASI-ASYMPTOTICALLY NONEXPANSIVE MAPPING IN BANACH SPACES*

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Abstract In this paper, we propose a new hybrid iterative scheme for finding a common solution of an equilibrium problem and fixed point of Bregman totally quasi-asymptotically nonexpansive mapping in reflexive Banach spaces. Moreover, we prove some strong convergence theorems under suitable control conditions. Finally, the application to zero point problem of maximal monotone operators is given by the result.

Key words equilibrium problem; Bregman totally quasi-asymptotically nonexpansive mapping; Bregman distance; reflexive Banach space

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1 Introduction

In 1994, Blum and Oettli [1] firstly studied the equilibrium problem:

$$\text{(EP)} \qquad \text{Finding } x \in C \text{ such that } g(x,y) \geq 0 \text{ for all } y \in C, \tag{1.1}$$

where $g : C \times C \to R$ is a bifunction, the set of solutions of problem (1.1) is denoted by (EP). Since then, various equilibrium problems were investigated. It is well known that equilibrium problems and their generalizations were important tools for solving problems arising in the fields of linear or nonlinear programming, variational inequalities, complementary problems, optimization problems, fixed point problems and were widely applied to physics, structural analysis, management science and economics [2–3] etc. One of the most important and interesting topics in the theory of equilibria was to develop efficient and implementable algorithms for solving equilibrium problems and their generalizations [2–4] etc. Since the equilibrium problems have very close connections with both the fixed point problems and the variational inequalities problems, finding the common elements of these problems drew many people's attention and became one of the hot topics in the related fields in the past few years [5–15] etc. But some methods that were proposed to the equilibrium problem is in the Hilbert spaces.

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In 1967, Bregman [16] discovered an effective technique for using of the so-called Bregman distance function $D_f(.)$ in the process of designing and analyzing feasibility and optimization algorithms. This opened a growing area of research in which Bregman's technique was applied in various ways in order to design and analyze not only iterative algorithms for solving feasibility and optimization problems, but also algorithms for solving variational inequalities, for approximating equilibria, for computing fixed point of nonlinear mapping and so on [6, 12, 13, 17, 18] etc.

We know that the fixed point theory of nonexpansive mappings can be applied to solutions of diverse problems such as finding zeroes of monotone mappings and solutions to certain evolution equations and solving convex feasibility, variational inequality and equilibrium problems. There are, in fact, many papers that deal with methods for finding fixed points of nonexpasive and quasi-nonexpansive mappings in Hilbert, uniformly convex and uniformly smooth Banach spaces [10, 20–29] etc.

When we try to extend this theory to general Banach spaces we encounter some difficulties, because many of the useful examples of nonexpansive mappings in Hilbert space are no longer nonexpansive in Banach spaces, an example of a Bregman nonexpansive mapping which is not a nonexpansive mapping in Hilbert space [30], for example,the resolvent $R_A := (I + A)^{-1}$ of a maximal monotone mapping $A : H \to 2^H$ and the metric projection P_C. There are several ways to overcome these difficulties. One of them is to use the Bregman distance instead of the norm, Bregman (quasi-) nonexpansive mappings instead of the (quasi-) nonexpansive mappings and the Bregman projection instead of the metric projection.

In 2014, Chang and Wang et al. [31] introduced the concept of Bregman totally quasi-asymptotically nonexpansive mapping and obtained the strong convergence of common fixed point for Bregman totally quasi-asymptotically nonexpansive mapping in reflexive Banach spaces.

In this paper, our results extend the one of Chang, Wang and Chan [31] to propose a new hybrid iterative scheme for finding a common solution of an equilibrium problem and fixed point of Bregman totally quasi-asymptotically nonexpansive mapping in reflexive Banach spaces. Moreover, we prove some strong convergence theorems under suitable control conditions. Finally, the application to zero point problem of maximal monotone operators is given by the result.

2 Preliminaries

Let E be a real Banach space, E^* be the dual space of E, $f : E \to (-\infty, +\infty]$ be a proper and real function, $f^* : E^* \to (-\infty, +\infty]$ be the Fenchel conjugate of f defined by

$$f^*(x^*) = \sup\{\langle x^*, x\rangle - f(x) : x \in E\}, x^* \in E^*.$$

We denote by dom$f$ the domain of f, that is, the set $\{x \in E : f(x) < \infty\}$ for any $x \in$ int(dom$f$) and $y \in E$, we define the right-hand derivative of f at x in the direction y by

$$f^0(x, y) = \lim_{t\to 0^+} \frac{f(x + ty) - f(x)}{t}.$$

The function f is said to be Gâteaux differentiable at x if $\lim\limits_{t\to 0^+} \frac{f(x+ty)-f(x)}{t}$ exists for any y.