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VISCOSITY APPROXIMATION METHODS FOR THE SPLIT EQUALITY COMMON FIXED POINT PROBLEM OF QUASI-NONEXPANSIVE OPERATORS*

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Abstract Let H_1, H_2, H_3 be real Hilbert spaces, let $A: H_1 \to H_3$, $B: H_2 \to H_3$ be two bounded linear operators. The split equality common fixed point problem (SECFP) in the infinite-dimensional Hilbert spaces introduced by Moudafi (Alternating CQ-algorithm for convex feasibility and split fixed-point problems. Journal of Nonlinear and Convex Analysis) is

$$\text{to find } \ x \in F(U), y \in F(T) \ \text{ such that } \ Ax = By, \tag{1}$$

where $U: H_1 \to H_1$ and $T: H_2 \to H_2$ are two nonlinear operators with nonempty fixed point sets $F(U) = \{x \in H_1 : Ux = x\}$ and $F(T) = \{x \in H_2 : Tx = x\}$. Note that, by taking $B = I$ and $H_2 = H_3$ in (1), we recover the split fixed point problem originally introduced in Censor and Segal. Recently, Moudafi introduced alternating CQ-algorithms and simultaneous iterative algorithms with weak convergence for the SECFP (1) of firmly quasi-nonexpansive operators. In this paper, we introduce two viscosity iterative algorithms for the SECFP (1) governed by the general class of quasi-nonexpansive operators. We prove the strong convergence of algorithms. Our results improve and extend previously discussed related problems and algorithms.

Key words split equality common fixed point problems; quasi-nonexpansive operator; strong convergence; viscosity iterative algorithms; Hilbert space

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1 Introduction

The split equality common fixed point problem (SECFP) was investigated recently, which is a generalization of the split feasibility problem and of the convex feasibility problem. The SECFP attracted many authors' attention due to its extraordinary utility and broad applicability in many areas of applied mathematics (most notably, fully-discretized models inverse problems which arise from phase retrievals and in medical image reconstruction [1]). Various algorithms were invented to solve it (see [2–6]). In this paper, our interest is in the study of the convergence of two viscosity iterative algorithms for the following SECFP introduced by Moudafi [7]:

$$\text{find } \ x \in F(U), y \in F(T) \ \text{ such that } \ Ax = By, \tag{1.1}$$

where $A : H_1 \to H_3$ and $B : H_2 \to H_3$ are two bounded linear operators, $U : H_1 \to H_1$ and $T : H_2 \to H_2$ are two nonlinear operators with nonempty fixed point sets $F(U) = C$ and $F(T) = Q$. This allows asymmetric and partial relations between the variables x and y. The interest is to cover many situations, for instance in decomposition methods for PDE's, applications in game theory and in intensity-modulated radiation therapy (IMRT). In decision sciences, this allows to consider agents who interact only via some components of their decision variables (see [8]). In IMRT, this amounts to envisage a weak coupling between the vector of doses absorbed in all voxels and that of the radiation intensity (see [9]).

To begin with, let us recall that the split feasibility problem (SFP) is to find a point

$$x \in C \ \text{ such that } \ Ax \in Q, \tag{1.2}$$

where C and Q are nonempty closed convex subset of real Hilbert spaces H_1 and H_2, respectively, and $A : H_1 \to H_2$ is a bounded linear operator. The SFP in finite-dimensional Hilbert spaces was originally introduced by Censor and Elfving [10]. Many algorithms were invented to solve it (see [3–5, 10–15] and references therein).

Note that if the split feasibility problem (1.2) is consistent (i.e., (1.2) has a solution), it is no hard to see that x^* solves the SFP (1.2) if and only if it solves the fixed point equation

$$P_C(I - \gamma A^*(I - P_Q)A)x^* = x^*, \tag{1.3}$$

where P_C and P_Q are the (orthogonal) projection onto C and Q, respectively, $\gamma > 0$ is any positive constant and A^* denotes the adjoint of A. This implies that we can use fixed point algorithms (see [12, 16, 17]) to solve SFP.

To solve (1.3), Byrne [1] proposed his CQ algorithm which generates a sequence $\{x_k\}$ by

$$x_{k+1} = P_C(I - \gamma A^*(I - P_Q)A)x_k, \ k \in N,$$

where $\gamma \in (0, \frac{2}{\lambda})$ with λ being the spectral radius of the operator A^*A.

Censor and Segal [18] considered the following split common fixed point problem (SCFP):

$$\text{find } \ x^* \in F(U) \ \text{ such that } \ Ax^* \in F(T), \tag{1.4}$$

where $A : H_1 \to H_2$ is a bounded linear operator, $U : H_1 \to H_1$ and $T : H_2 \to H_2$ are two nonexpansive operators with nonempty fixed point sets $F(U) = C$ and $F(T) = Q$.

To solve (1.4), Censor and Segal [18] proposed and proved, in finite-dimensional spaces, the convergence of the following algorithm:

$$x_{k+1} = U(x_k + \gamma A^t(T - I)Ax_k), \quad k \in N,$$