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EXISTENCE OF SOLUTION AND APPROXIMATE CONTROLLABILITY OF A SECOND-ORDER NEUTRAL STOCHASTIC DIFFERENTIAL EQUATION WITH STATE DEPENDENT DELAY*

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Abstract This paper has two sections which deals with a second order stochastic neutral partial differential equation with state dependent delay. In the first section the existence and uniqueness of mild solution is obtained by use of measure of non-compactness. In the second section the conditions for approximate controllability are investigated for the distributed second order neutral stochastic differential system with respect to the approximate controllability of the corresponding linear system in a Hilbert space. Our method is an extension of co-author N. Sukavanam's novel approach in [22]. Thereby, we remove the need to assume the invertibility of a controllability operator used by authors in [5], which fails to exist in infinite dimensional spaces if the associated semigroup is compact. Our approach also removes the need to check the invertibility of the controllability Gramian operator and associated limit condition used by the authors in [20], which are practically difficult to verify and apply. An example is provided to illustrate the presented theory.

Key words approximate controllability; cosine family; state dependent delay; neutral stochastic differential equation; measure of noncompactness

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1 Introduction

Random noise causes fluctuations in deterministic models. So, necessarily we move from deterministic problems to stochastic ones. Stochastic evolution equations are natural generalizations of ordinary differential equations incorporating the randomness into the equations. Thereby, making the system more realistic. [7–17] and the references therein explore the qualitative properties of solutions for stochastic differential equations. Considering the environmental disturbances, Kolmanovskii and Myshkis [18] introduced a class of neutral stochastic functional differential equations, which are applicable in several fields such as chemical engineering, aeroelasticity and so on. In recent years, controllability of stochastic infinitedimensional systems was

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extensively studied for various applications. Several papers studied the approximate controllability of semilinear stochastic control systems, see for instance [5, 8, 9, 19, 20] and references therein. Controllability results are available in overwhelming majority for abstract stochastic differential delay systems; rather than for neutral second-order stochastic differential with state dependent delay.

Mahmudov [20] investigated conditions on the system operators so that the semilinear control system is approximately controllable provided the corresponding linear system is approximately controllable. The main drawback of the papers [8, 19, 20] is the need to check the invertibility of the controllability Gramian operator and a associated limit condition, which are practically difficult to verify and apply.

Neutral differential equations appear in several areas of applied mathematics, and thus studied in several papers and monographs, see for instance [11, 12, 23] and references therein. Differential equations with delay reflect physical phenomena more realistically than those without delay.

Recently, much attention was paid to partial functional differential equation with state dependent delay. For details see [1, 3, 13–15]. As a matter of fact, in these papers their authors assume severe conditions on the operator family generated by A, which imply that the underlying space X has finite dimension. Thus the equations treated in these works are really ordinary and not partial equations.

Our method builds on co-author Sukavanam's novel approach in [22]. We also remove the need to assume the invertibility of a controllability operator used by authors in [4–6, 21] which fails to exist in infinite dimensional spaces if the associated semigroup is compact. Our approach also removes the drawbacks of the method applied in [8, 19, 20].

Hence motivated by this fact in this paper we study the existence and uniqueness of mild solution and approximate controllability of the partial neutral stochastic differential equation of second order with state delay. Specifically we study the following second order equations modelled in the form

$$\begin{aligned} \mathrm{d}(x'(t)+g(t,x_t)) &= [Ax(t)+f(t,x_{\rho(t,x_t)})+Bu(t)]\mathrm{d}t \\ &\quad +G(t,x_t)\mathrm{d}W(t) \ \ \text{a.e. on } t\in J=[0,a], \\ x_0=\phi\in\mathfrak{B},\ \ x'(0)&=\psi\in X, \end{aligned} \tag{1.1}$$

where A is the infinitesimal generator of a strongly continuous cosine family $\{C(t) : t \in \mathsf{R}\}$ of bounded linear operators on a Hilbert space X. Let $(\Omega, \mathcal{F}, P)$ be a probability space together with a normal filtration $\mathcal{F}_t,\ t \geq 0$. The state space $x(t) \in X$ and the control $u(t) \in L_2^{\mathcal{F}}(J, U)$, where X and U are separable Hilbert spaces and d is the stochastic differentiation. The history valued function $x_t : (-\infty, 0] \to X$, $x_t(\theta) = x(t + \theta)$ belongs to some abstract phase space $\mathfrak{B}$ defined axiomatically; g, f are appropriate functions; B is a bounded linear operator on a Hilbert space U. Let K be a separable Hilbert space and $\{W(t)\}_{t\geq0}$ is a given K-valued Brownian motion or Wiener process with finite trace nuclear covariance operator $Q > 0$. The functions $f, g : J \times \mathfrak{B} \to X$ are measurable mappings in X norm and $G : J \times \mathfrak{B} \to L_Q(K, X)$ is a measurable mapping in $L_Q(J, X)$ norm. $L_Q(J, X)$ is the space of all Q-Hilbert Schmidt operators from K into X; B is a bounded linear operator from U into X; $\phi(t)$ is $\mathfrak{B}$-valued random variable independent of Brownian motion $W(t)$ with finite second moment. Also $\psi(t)$