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GROWTH OF MEROMORPHIC SOLUTIONS OF SOME ALGEBRAIC DIFFERENTIAL EQUATIONS*

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Abstract In this paper, by means of the normal family theory, we estimate the growth order of meromorphic solutions of some algebraic differential equations and improve the related result of Barsegian et al. [6]. We also give some examples to show that our results occur in some special cases.

Key words the normal family theory; algebraic differential equations; meromorphic solutions; growth

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1 Introduction and Main Results

Let $f(z)$ be a function meromorphic or holomorphic in the complex plane. We use the standard notations of Nevanlinna theory and denote the order of $f(z)$ by $\rho(f)$ (see Hayman [13], He [14], Laine [15] and Yang [17]).

Let D be a domain in the complex plane. A family $\mathcal{F}$ of meromorphic functions in D is normal, if each sequence $\{f_n\} \subset \mathcal{F}$ contains a subsequence which converges locally uniformly by spherical distance to a function $g(z)$ meromorphic in D ($g(z)$ is permitted to be identically infinity).

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We define spherical derivative of the meromorphic function $f(z)$ as follows:

$$f^{\sharp}(z) := \frac{|f'(z)|}{1+|f(z)|^2}.$$

An algebraic differential equation for $w(z)$ is of the form

$$P(z, w, w', \cdots, w^{(k)}) = 0, \tag{1.1}$$

where P is a polynomial in each of its variables.

It is one of the important and interesting subjects to research the growth of meromorphic solution $w(z)$ of differential equation (1.1) in the complex plane.

In 1956, Gol′dberg [9] proved that the meromorphic solutions have finite growth order when $k = 1$. Some alternative proofs of this result were given by Bank and Kaufman [1], by Barsegian [2].

In 1998, Barsegian (see [4] and [6]) introduced an essentially new type of weight for differential monomial below and gave the estimate first time for the growth order of meromorphic solutions of large classes of complex differential equations of higher degrees by using his initial method [5]. Later Bergweiler [7] reproved Barsegian's result by using Zalcman's lemma.

In order to state the result, we first introduce some notations [4]: $n \in \mathrm{N} = \{1, 2, 3, \cdots\}, t_j \in \mathrm{N}_0 = \mathrm{N}\bigcup\{0\}$ for $j = 1, 2, \cdots, n$, and put $t = (t_1, t_2, \cdots, t_n)$. Define $M_t[w]$ by

$$M_t[w](z) := [w'(z)]^{t_1}[w''(z)]^{t_2} \cdots [w^{(n)}]^{t_n}$$

with the convention that $M_{\{0\}}[w] = 1$. We call $p(t) := t_1 + 2t_2 + \cdots + nt_n$ the weight of $M_t[w]$. A differential polynomial $P[w]$ is an expression of the form

$$P[w](z) := \sum_{t\in I} a_t(z, w(z))M_t[w](z), \tag{1.2}$$

where the a_t are rational in two variables and I is a finite index set. The total weight $W(P)$ of $P[w]$ is given by $W(P) := \max\limits_{t\in I} p(t)$.

Definition 1.1 $\deg_{z,\infty} a_t$ denotes the degree at infinity in variable z concerning $a_t(z, w)$. $\deg_{z,\infty} a := \max\limits_{t\in I} \max\{\deg_{z,\infty} a_t, 0\}$,

$$\alpha_{m,P} := \max_{t\in I, m>p(t)} \frac{\max\{\deg_{z,\infty} a_t, 0\}}{m - p(t)},$$

$$\beta_{m,P} := \max_{t\in I, m=p(t)} \deg_{z,\infty} a_t.$$

When all $p(t) = m,\ t \in I$, we set $\alpha_{m,P} = 0$.

In 2002, the following general estimate of growth order of meromorphic solutions $w(z)$ of the equation $[w'(z)]^m = P[w]$ was obtained, which depend on the degrees at infinity of coefficients of differential polynomial in z, by Barsegian et al. [6].

Theorem 1.2 (see [6]) Let $w(z)$ be a meromorphic solution to the differential equation $[w'(z)]^m = P[w]$ where $m \in \mathrm{N}$. If $m > W(P)$ or $m = W(P)$ and $\beta_{m,P} < 0$, then the growth order $\rho := \rho(w)$ of $w(z)$ satisfies $\rho \leq 2 + 2\alpha_{m,P}$.

Remark 1.3 Barsegian [3, 4], Bergweiler [7], Frank and Wang [12] and Yuan et al. [19, 20] proved $\rho < \infty$ or the conditions hold for all $t \in I$.