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CONTROLLABILITY AND OPTIMALITY OF LINEAR TIME-INVARIANT NEUTRAL CONTROL SYSTEMS WITH DIFFERENT FRACTIONAL ORDERS*

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Abstract Control systems governed by linear time-invariant neutral equations with different fractional orders are considered. Sufficient and necessary conditions for the controllability of those systems are established. The existence of optimal controls for the systems is given. Finally, two examples are provided to show the application of our results.

Key words controllability; optimality; neutral equations; different fractional orders

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1 Introduction

Fractional calculus was proved to be useful tools in the investigation of many phenomena in engineering, physics, economy, chemistry, electrodynamics of complex medium and other fields, see for instance [1–5]. Many authors investigated the problem for different types of fractional differential equations, including fractional ordinary differential equations, fractional functional differential equations, fractional partial differential equations, and so on.

As one of the important topics in mathematical control theory, controllability plays an important role in the analysis and design of control systems. In recent years, the problem of controllability for various kinds of fractional differential equations was extensively studied by many researchers [6–15]. The main tool employed in the analysis of controllability is based on

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an application of the fixed point theorem. However, the controllability results obtained by the fixed point theorem are inconvenient for computation in practical problems.

Inspired by the above-mentioned works, in this article, we follow the ideas in [9] to investigate the controllability of linear time-invariant neutral control systems with different fractional orders, and then give the optimality of such control systems. More precisely, we consider the following linear time-invariant neutral control system:

$$\begin{cases} {}_0^C D_t^{\alpha} x(t) = Ax(t) + Bx(t-r) + N \cdot {}_0^C D_t^{\beta} x(t-r) + Gu(t), \quad t \geq 0, \\ y(t) = Ex(t) + Fu(t), \\ x(t) = \phi(t), \quad -r \leq t \leq 0, \end{cases} \tag{1.1}$$

where $A, B, N \in \mathbb{R}^{n\times n}$, $G \in \mathbb{R}^{n\times p}$, $E \in \mathbb{R}^{k\times n}$, $F \in \mathbb{R}^{k\times p}$, $0 < \beta < \alpha < 1$, state variable $x(t) \in \mathbb{R}^n$, initial function $\phi(t) \in C([-r, 0], \mathbb{R}^n)$, and control input $u(t) \in \mathbb{R}^p$. In fact, differential equations with different fractional orders often arise in many physical and engineering models, see [16–25]. So it is necessary to study the controllability and optimality of the neutral control systems with different fractional orders.

This article is organized as follows. In Section 2, we briefly review the definitions of fractional calculus. We present the concept of controllability for the linear time-invariant neutral control systems with different fractional orders. In Section 3, we give the sufficient and necessary conditions for the controllability of the control systems. In Section 4, we discuss the existence of optimal control. In Section 5, the applicability of the presented theory is demonstrated with two examples, and Section 6 briefly summarizes the results of this article.

2 Preliminaries

In this section, we give some basic definitions and results that are used throughout this article. For more details, please see [26, 27].

Definition 2.1 Let $[a, b]$ be a finite interval on the real axis $\mathbb{R}$. The fractional integral of order $\alpha > 0$ with the lower limit a for the function x is defined as

$$({}_a I_t^{\alpha} x)(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} x(\tau) \mathrm{d}\tau, \quad a < t \leq b$$

provided the right-hand side is pointwise defined on $[a, b]$, where $\Gamma(\cdot)$ is the Gamma function.

Definition 2.2 Let $[a, b]$ be a finite interval on the real axis $\mathbb{R}$, $n-1 < \alpha \leq n$, $n \in \mathbb{N}^+$. And let the function $x(t)$ have continuous derivatives up to order n such that $x^{(n)}(t)$ is absolutely continuous on $[a, b]$. The Caputo fractional derivative $({}_a^C D_t^{\alpha} x)(t)$ of order α is defined as

$$({}_a^C D_t^{\alpha} x)(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-\tau)^{n-\alpha-1} x^{(n)}(\tau) \mathrm{d}\tau, \quad a < t \leq b.$$

The Laplace transform of the Caputo's fractional derivative $({}_0^C D_t^{\alpha} x)(t)$ is

$$\mathcal{L}\{({}_0^C D_t^{\alpha} x)(t); s\} = s^{\alpha} \mathcal{L}\{x(t); s\} - \sum_{i=0}^{n-1} s^{\alpha-i-1} x^{(i)}(0^+), \quad t > 0.$$

Definition 2.3 The two-parameter Mittag-Leffler function is defined as

$$E_{\alpha,\beta}(z) = \sum_{i=0}^{+\infty} \frac{z^i}{\Gamma(\alpha i + \beta)}, \quad \alpha, \beta > 0, z \in \mathbb{C}. \tag{2.1}$$