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MULTIPLICITY RESULTS FOR FOURTH ORDER ELLIPTIC EQUATIONS OF KIRCHHOFF-TYPE*

Liping XU (许丽萍)
Department of Mathematics and Statistics, Henan University of Science and Technology, Luoyang 471003, China
E-mail: *x.liping@126.com*

Haibo CHEN (陈海波)†
School of Mathematics and Statistics, Central South University, Changsha 410075, China
E-mail: *math_chb@csu.edu.cn*

Abstract In this paper, we concern with the following fourth order elliptic equations of Kirchhoff type

$$\begin{cases} \triangle^2 u - \left(a + b\displaystyle\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)\triangle u + V(x)u = f(x,u), x \in \mathbb{R}^3, \\ u \in H^2(\mathbb{R}^3), \end{cases}$$

where $a, b > 0$ are constants and the primitive of the nonlinearity f is of superlinear growth near infinity in u and is also allowed to be sign-changing. By using variational methods, we establish the existence and multiplicity of solutions. Our conditions weaken the Ambrosetti-Rabinowitz type condition.

Key words fourth order elliptic equations of Kirchhoff type; symmetric mountain pass theorem; variational methods

2010 MR Subject Classification 35J20; 35J65; 35J60

1 Introduction

Consider the following fourth order elliptic equations of Kirchhoff type

$$\begin{cases} \triangle^2 u - \left(a + b\displaystyle\int_{\mathbb{R}^3} |\nabla u|^2 \mathrm{d}x\right)\triangle u + V(x)u = f(x,u), x \in \mathbb{R}^3, \\ u \in H^2(\mathbb{R}^3), \end{cases} \tag{1.1}$$

where a, b are positive constants. We assume that the functions $V(x)$, $f(x,u)$ and $F(x,u) = \int_0^u f(x,s)\mathrm{d}s$ satisfy the following hypotheses.

(V) $V(x) \in C(\mathbb{R}^3, R)$ satisfies $\inf\limits_{x\in\mathbb{R}^3} V(x) \geq a_1 > 0$, where $a_1 > 0$ is a constant. Moreover, for any $M > 0$, $\mathrm{meas}\{x \in \mathbb{R}^3 : V(x) \leq M\} < \infty$, where meas(.) denotes the Lebesgue measure in $\mathbb{R}^3$.

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†Corresponding author: Haibo CHEN.

$(f1)$ $f(x,u)\in C(\mathbb{R}^3\times\mathbb{R},\mathbb{R})$ and there exist $\alpha,\beta>0$ such that

$$|f(x,u)|\le \alpha|u|+\beta|u|^{p-1} \text{ for some } p\in(2,6).$$

$(f2)$ $\lim\limits_{|u|\to+\infty}\frac{|F(x,u)|}{|u|^4}=\infty$ a.e. $x\in\mathbb{R}^3$ and there exists $\gamma>0$ such that

$$F(x,u)\ge 0,\ \forall (x,u)\in\mathbb{R}^3\times\mathbb{R},\ |u|\ge\gamma.$$

$(f3)$ Set $\tilde{F}(x,u)=\frac{1}{4}uf(x,u)-F(x,u)\ge 0$ and there exist $k>\frac{3}{2},\nu>0$ such that

$$|F(x,u)|^k\le \nu|u|^{2k}\tilde{F}(x,u),\ \forall (x,u)\in\mathbb{R}^3\times\mathbb{R},|u|\ge\gamma.$$

$(f4)$ There exist $\mu>4$ and $\rho>0$ such that

$$\mu F(x,u)\le uf(x,u)+\rho u^2,\ \forall (x,u)\in\mathbb{R}^3\times\mathbb{R}.$$

$(f5)$ $f(x,-u)=-f(x,u),\ \forall (x,u)\in\mathbb{R}^3\times\mathbb{R}$.

$(f6)$ There exist $\mu>4$ and $\gamma_1>0$ such that

$$\mu F(x,u)\le uf(x,u),\ \forall (x,u)\in\mathbb{R}^3\times\mathbb{R},\ |u|\ge\gamma_1.$$

Problem (1.1) is a nonlocal problem because of the appearance of the term $\int_{\mathbb{R}^3}|\nabla u|^2\mathrm{d}x$ which provokes some mathematical difficulties. This makes the study of (1.1) particularly interesting. Let $V(x)=0$, replace $\mathbb{R}^3$ by a bounded smooth domain $\Omega\subset\mathbb{R}^N$ and set $u=\triangle u=0$ on $\partial\Omega$, then problem (1.1) is reduced the following fourth order elliptic equations of Kirchhoff type

$$\begin{cases}\triangle^2u-\left(a+b\displaystyle\int_\Omega|\nabla u|^2\mathrm{d}x\right)\triangle u=f(x,u), & x\in\Omega,\\ u=0,\triangle u=0 \text{ on } \partial\Omega,\end{cases}\tag{1.2}$$

which is related to the following stationary analogue of the equation of Kirchhoff type

$$u_{tt}+\triangle^2u-\left(a+b\int_\Omega|\nabla u|^2\mathrm{d}x\right)\triangle u=f(x,u),\quad \text{in } \Omega,\tag{1.3}$$

where $\triangle^2$ is the biharmonic operator. In one and two dimensions, (1.3) is used to describe some phenomena appeared in different physical, engineering and other sciences because it is regarded as a good approximation for describing nonlinear vibrations of beams or plates (see [1, 2]). In [3], Ma et al. considered existence and multiplicity of positive solutions for the fourth order equation

$$\begin{cases}u''''-M\left(\displaystyle\int_0^1|u'|^2\mathrm{d}x\right)u''=q(x)f(x,u,u'),\\ u(0)=u(1)=u''(0)=u''(1)=0\end{cases}\tag{1.4}$$

by using the fixed point theorems in cones of ordered Banach spaces. Recently, by the variational methods, Ma and Wang etc. studied (1.4) and the following fourth order equation of Kirchhoff type

$$\begin{cases}\triangle^2u-M\left(\displaystyle\int_\Omega|\nabla u|^2\mathrm{d}x\right)\nabla u=f(x,u) \text{ in } \Omega,\\ u=\triangle u=0 \text{ on } \partial\Omega\end{cases}$$

and obtained the existence and multiplicity of solutions, see [4–6]. Very recently, Wang et al. considered the existence of nontrivial solutions for (1.2) with one parameter λ in [7] by using

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