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# EVOLUTION FILTRATION PROBLEMS WITH SEAWATER INTRUSION: TWO-PHASE FLOW DUAL MIXED VARIATIONAL ANALYSIS*

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**Abstract** Tow-phase flow mixed variational formulations of evolution filtration problems with seawater intrusion are analyzed. A dual mixed fractional flow velocity-pressure model is considered with an air-fresh water and a fresh water-seawater characterization. For analysis and computational purposes, spatial decompositions based on nonoverlapping multidomains, above and below the sea level, are variationally introduced with internal boundary fluxes dualized as weak transmission constraints. Further, parallel augmented and exactly penalized duality algorithms, and proximation semi-implicit time marching schemes, are established and analyzed.

**Key words** two-phase flow in coastal aquifers; fractional two-phase flow; dual mixed variational analysis; macro-hybrid variational formulations; augmented exactly penalized duality algorithms; proximation semi-implicit time marching schemes

**2010 MR Subject Classification** 35A15; 47H05; 35J50; 76S05; 35R35

## 1 Introduction

The purpose of this paper is to formulate and analyze variational two-phase flows in open coastal aquifers. For the physical model, we adopt the fractional two-phase dual mixed model of Chen and Ewing [1, 2], with an air-fresh water and a fresh water-seawater characterization, assuming the fresh water velocity as the wetting phase velocity. This mixed flow model corresponds to an instantaneous total velocity-global pressure incompressible flow, coupled with an evolution wetting velocity-complementary pressure compressible-like flow. Importantly, such a dual mixed two-phase flow modeling is appropriate for the application of composition duality methods [3, 4], in the solvability analysis of the system via duality principles. For analysis and computational purposes, suitable spatial decompositions, based on nonoverlapping multidomains, above and below the sea level, were variationally introduced with internal boundary fluxes dualized as weak transmission macro-hybrid constraints [5–7]. We should emphasize that these decomposed reformulations lead to macro-hybrid mixed localized models, which are very

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appropriate for internal variational finite dimensional approximations, implementable in terms of non-matching finite element discretizations [8, 9].

Further, parallel augmented and exactly penalized duality algorithms, as well as proximation semi-implicit time marching schemes of the Douglas-Rachford and Peaceman-Rachford type, are established and analyzed.

In a recent study [10], we considered this evolution filtration problem in the sense of a slightly compressible Darcian velocity-pressure mixed single phase flow, which corresponds to a mixed variational approach of the classical variational pressure model presented in [11]. Such a classical pressure model was in turn an extension to general domains with seawater intrusion, of evolution filtration problems based on the pioneer Baiochi's transform analysis [12]; see [13–17]. In this paper, our interest is to apply a special two-phase approach to the evolution coastal filtration problem, in conjunction with a natural compositional duality macro-hybrid mixed variational analysis.

## 2 Physical and Mixed Variational Models

In this section, we start with a qualitative description of the open coastal aquifer to be considered, and the definition of its two-phase flow model. Corresponding mixed variational formulations are established through composition duality methods, and their solvability analysis is performed via duality principles.

### 2.1 The Qualitative Mixed Physical Model

Let $\Omega \subset \Re^3$ denote the spatial configuration of a coastal aquifer, a bounded connected domain with a Lipschitz continuous boundary $\partial\Omega$. We shall consider the following partition of boundary $\partial\Omega$ (Figure 1):

$$\begin{aligned} \partial\Omega_i &= \text{the impervious part,} \\ \partial\Omega_{fw} &= \text{the part in contact with fresh water,} \\ \partial\Omega_{sw} &= \text{the part in contact with seawater,} \\ \partial\Omega_a &= \text{the part in contact with open air.} \end{aligned} \tag{2.1}$$

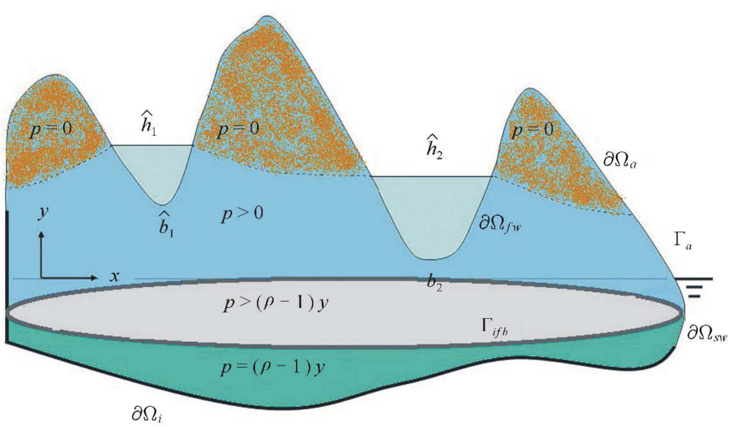


Fig.1 A section of an open coastal aquifer