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WEAK TYPE INEQUALITY FOR THE MAXIMAL OPERATOR OF WALSH-KACZMARZ-MARCINKIEWICZ MEANS*

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Abstract The main aim of this article is to prove that the maximal operator σ_*^{κ} of the Marcinkiewicz-Fejér means of the two-dimensional Fourier series with respect to Walsh-Kaczmarz system is bounded from the Hardy space $H_{2/3}$ to the space weak-$L_{2/3}$.

Key words Walsh-Kaczmarz system, Marcinkiewicz-Fejér means, Maximal operator, a.e. convergence, weak type inequality

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1 Introduction

For the two-dimensional Walsh-Fourier series, Weisz [1] proved that the maximal operator

$$\sigma_*^w f = \sup_{n\in\mathbf{P}} \frac{1}{n}\left|\sum_{j=0}^{n-1} S_{j,j}^w (f)\right|$$

is bounded from the two-dimensional dyadic martingale Hardy space H_p to the space L_p for $p > 2/3$. The d-dimensional version of this result was reached in [2]. In [3] the author proved that in theorem of Weisz the assumption $p > 2/3$ is essential; in particular it was showed that the maximal operator σ_*^w is not bounded from the Hardy space $H_{2/3}$ to the space $L_{2/3}$. On the other hand, it is proved that in the endpoint case $p = 2/3$, the maximal operator of the Marcinkiewicz-Fejér means of the double Walsh-Fourier series is bounded from the dyadic Hardy space $H_{2/3}$ to the space weak-$L_{2/3}$ [4].

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It is well-known that $\varepsilon_n \left|D_n^w(x)\right| \to 0$ as $n \to \infty$ [5] for every $x \in K$. The Dirichlet kernels belonging to the Kaczmarz ordering do not satisfy this property [6]. Namely, in 1948 Šneider [6] introduced the Walsh-Kaczmarz system and showed that the inequality

$$\limsup_{n\to\infty} \frac{D_n^\kappa(x)}{\log n} \ge C > 0$$

holds a.e. Consequently, it is harder to obtain pointwise convergence results for Walsh-Kaczmarz-Fourier series than for Walsh-Fourier series. In 1974, Schipp [7] and Young [8] proved that the Walsh-Kaczmarz system is a convergence system. In 1981, Skvortsov [9] showed that the Fejér means with respect to the Walsh-Kaczmarz system converge uniformly to f for any continuous functions f. Gát [10] proved, for any integrable functions, that the Fejér means with respect to the Walsh-Kaczmarz system converge almost everywhere to the function itself. Gát's theorem was extended by Simon [11] to H_p spaces. Namely, he proved that the maximal operator of Fejér means of one-dimensional Fourier series is bounded from dyadic martingale Hardy space H_p to the space L_p for $p > 1/2$. In the end point case $p = 1/2$, the first author proved that the maximal operator is not bounded from dyadic martingale Hardy space $H_{1/2}$ to the space $L_{1/2}$ [12]. The case $0 < p < 1/2$ can be found in the papers of Tephnadze [13, 14].

In 2006, the almost everywhere convergence of the Marcinkiewicz-Fejér means of two-dimensional Walsh-Kaczmarz-Fourier series was showed [15]. Moreover, the second author discussed the properties of the maximal operator σ_*^κ, that is, the maximal operator is of weak type (1,1) and of type (p, p) for all $1 < p \le \infty$. In [16, 17], it was proven that the maximal operator σ_*^κ is bounded from the dyadic Hardy-Lorentz space H_p into L_p space for every $p > 2/3$. We also proved that the assumption $p > 2/3$ is essential [18]; in particular, we showed that the maximal operator σ_*^κ is not bounded from the Hardy space $H_{2/3}$ to the space $L_{2/3}$.

In this article, we generalize the results of [15–17] for maximal operator σ_*^κ and prove that the maximal operator σ_*^κ of the Marcinkiewicz-Fejér means with respect to the Walsh-Kaczmarz system is bounded from the Hardy space $H_{2/3}$ to the space weak-$L_{2/3}$.

2 Definitions and Notations

Let $\mathbf{P}$ denote the set of positive integers, $\mathbf{N} := \mathbf{P} \cup \{0\}$. Denote Z_2 the discrete cyclic group of order 2, that is $Z_2 = \{0, 1\}$, where the group operation is the modulo 2 addition and every subset is open. The Haar measure on Z_2 is given such that the measure of a singleton is $1/2$. Let K be the complete direct product of the countable infinite copies of the compact groups Z_2. The elements of K are of the form $x = (x_0, x_1, \cdots, x_k, \cdots)$ with coordinates $x_k \in \{0, 1\}\,(k \in \mathbf{N})$. The group operation on K is the coordinate-wise addition, and the measure (denoted by μ) and the topology are the product measure and topology. The compact Abelian group K is called the Walsh group. A base for the neighbourhoods of K can be given in the following way [5, 19]:

$$\begin{aligned} &I_0(x) := K, \\ &I_n(x) := I_n(x_0, \cdots, x_{n-1}) := \{y \in K :\ y = (x_0, \cdots, x_{n-1}, y_n, y_{n+1}, \cdots)\}, \\ &\qquad\qquad (x \in K, n \in \mathbf{N}). \end{aligned}$$

These sets are called dyadic intervals. Let $0 = (0 : i \in \mathbf{N}) \in K$ denote the null element of K, $I_n := I_n(0)$ $(n \in \mathbf{N})$. Set $e_n := (0, \cdots, 0, 1, 0, \cdots) \in K$, the nth coordinate of which is 1