



MULTIPLICITY OF SOLUTIONS FOR A QUASILINEAR ELLIPTIC EQUATION*



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Abstract We study a quasilinear elliptic equation with polynomial growth coefficients. The existence of infinitely many solutions is obtained by a dual method and a nonsmooth critical point theory.

Key words Quasilinear equation; dual method; critical point; nonsmooth analysis

2010 MR Subject Classification 35J20; 35J62; 49J52

1 Introduction and Main Result

Consider the following quasilinear elliptic equation of the form

$$-\sum_{i,j=1}^N D_j(a_{ij}(u)D_i u) + \frac{1}{2} \sum_{i,j=1}^N a'_{ij}(u)D_i u D_j u + V(x)u = f(u), \quad x \in \mathbb{R}^N, \quad (1.1)$$

where $N \geq 3$, $D_i = \frac{\partial}{\partial x_i}$, and $a'_{ij}(s) = \frac{d}{ds}a_{ij}(s)$. This equation includes the well-known Modified Nonlinear Schrödinger Equation(MNLSE)

$$-\Delta u + V(x)u - \Delta(u^2)u = f(u), \quad x \in \mathbb{R}^N. \quad (1.2)$$

Indeed, equation (1.2) is the special case of (1.1) with $a_{ij}(s) = (1 + 2s^2)\delta_{ij}$. This type of quasilinear equations appears in some models in mathematical physics (see [2, 3, 8]).

For the case in which some of coefficients a_{ij} in (1.1) are unbounded, the existence and multiplicity of solutions to (1.1) have been studied by several authors in recent years. For example, the existence of multibump solutions was obtained in [9, 10] by using a cutoff technique and nonsmooth critical point theory. The Nehari method was applied to establish the existence of both one-sign and nodal ground states of solutions in [12]. In [13, 15], by considering a 4-Laplacian perturbation equation of (1.1), solutions of (1.1) were obtained as limits of solutions of the perturbation problem.

In form, (1.1) has a energy functional

$$J(u) = \frac{1}{2} \int_{\mathbb{R}^N} \left[\sum_{i,j=1}^N a_{ij}(u)D_i u D_j u + V(x)u^2 \right] dx - \int_{\mathbb{R}^N} F(u)dx$$

*Received November 3, 2014; revised October 18, 2015. This work is supported in part by the National Natural Science Foundation of China (11261070).

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defined on the Hilbert space

$$H_V^1 = \left\{ u \in H^1(\mathbb{R}^N) : \int_{\mathbb{R}^N} V(x)u^2 dx < \infty \right\},$$

where $F(u) = \int_0^u f(s)ds$.

If $\sum_{i,j=1}^N a_{ij}(u)D_i u D_j u \geq cu^2|\nabla u|^2$ for some $c > 0$, there are some technical difficulties in applying variational methods directly to the functional J for a lack of an appropriate working space. It seems that there is no natural space in which the functional J possesses both smoothness and compactness properties. Indeed, if $J \in C^1(E, \mathbb{R})$ for some Banach space E , then E is smaller than $H^1(\mathbb{R}^N)$ because $\int_{\mathbb{R}^N} u^2 |\nabla u|^2 < \infty$ in E . It then seems impossible to verify the mountain pass geometry of the functional J and to obtain the bounds of a (PS) sequence. There are three main ideas to overcome these difficulties. One is applications of various variational techniques without the (PS) condition, such as the Nehari method[12]. Another is transformation methods in which the verification of the mountain pass geometry and the (PS) condition is available, such as the perturbation([13, 15]) method and the dual approach([5, 11]). A third is the nonsmooth critical point theory([9, 10]).

In this article, we transform equation (1.1) to another with a continuous energy functional in some Banach space. Nonsmooth critical point theory then can be applied to obtain the existence of multiple solutions to the original problem. Though the main ideas used here are originally due to M. Colin, L. Jeanjean[5] and J. Liu, Y. Wang and Z.Q. Wang[11], we do not use them directly but make some crucial modifications and use some new techniques.

We are interested in the following cases.

(V) V is Hölder continuous, $V(x) > 0$, and $\lim_{|x| \rightarrow \infty} V(x) = \infty$.

(f_1) f is odd and Hölder continuous.

(f_2) There exist constants $C > 0$, and $4 < p < 22^*$ such that

$$|f(s)| \leq C(1 + |s|^{p-1})$$

for all $s \in \mathbb{R}$, where $2^* = \frac{2N}{N-2}$ is the Sobolev critical exponent.

(f_3) $\frac{f(s)}{s} \rightarrow 0$ as $s \rightarrow 0$.

(f_4) There exist constants $\mu > 4$ and $r > 0$ such that $sf(s) - \mu F(s) > 0$ for $|s| > r$, where $F(s) = \int_0^s f(t)dt$.

(a_1) $a_{ij} \in C^{1,\gamma}(\mathbb{R}, \mathbb{R})$ for some $0 < \gamma < 1$, $a_{ij} = a_{ji}$, $a_{ij}(-s) = a_{ij}(s)$ and there exists a positive constant c_0 such that $|a'_{ij}(s)| \leq c_0^{-1}(1 + |s|)$ and

$$\sum_{i,j=1}^N a_{ij}(s)\xi_i \xi_j \geq c_0(1 + s^2)|\xi|^2$$

for $s \in \mathbb{R}$ and $\xi \in \mathbb{R}^N$.

(a_2) There exists a constant $\alpha > 0$ such that

$$0 \leq \sum_{i,j=1}^N sa'_{ij}(s)\xi_i \xi_j \leq (\mu - 2 - \alpha) \sum_{i,j=1}^N a_{ij}(s)\xi_i \xi_j$$

for $s \in \mathbb{R}$ and $\xi \in \mathbb{R}^N$.

We can state our main result now.

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