Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2016,**36B**(2):602–613
Acta Mathematica Scientia 数学物理学报
http://actams.wipm.ac.cn

GENERAL SPLIT FEASIBILITY PROBLEMS FOR TWO FAMILIES OF NONEXPANSIVE MAPPINGS IN HILBERT SPACES*

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Abstract The purpose of this article is to introduce a general split feasibility problems for two families of nonexpansive mappings in Hilbert spaces. We prove that the sequence generated by the proposed new algorithm converges strongly to a solution of the general split feasibility problem. Our results extend and improve some recent known results.

Key words General split feasibility problems; nonexpansive mappings; Hilbert space; strong convergence

2010 MR Subject Classification 90C25; 47H09; 47J25

1 Introduction

Let H and K be infinite-dimensional real Hilbert spaces, and let $A : H \to K$ be a bounded linear operator. Let $\{C_i\}_{i=1}^m$ and $\{Q_i\}_{i=1}^n$ be the families of nonempty closed convex subsets of H and K, respectively. Let $F(T)$ be the fixed point of the mapping T.

(a) The convex feasibility problem (CFP) is formulated as the problem of finding a point x^* with the property:

$$x^* \in \bigcap_{i=1}^{m} C_i. \tag{1.1}$$

*Received December 15, 2014; revised July 11, 2015. Supported by the Scientific Research Fund of Sichuan Provincial Department of Science and Technology (2015JY0165, 2011JYZ011), the Scientific Research Fund of Sichuan Provincial Education Department (14ZA0271), the Scientific Research Project of Yibin University (2013YY06), the Natural Science Foundation of China Medical University, Taiwan, and the National Natural Science Foundation of China (11361070).

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(b) The split feasibility problem (SFP) is formulated as the problem of finding a point x^* with the property:

$$x^* \in C, \ Ax^* \in Q, \tag{1.2}$$

where C and Q are nonempty, closed and convex subsets of H and K, respectively.

(c) The multiple-set split feasibility problem (MSSFP) is formulated as the problem of finding a point x^* with the property:

$$x^* \in \bigcap_{i=1}^{m} C_i, \ Ax^* \in \bigcap_{i=1}^{n} Q_i. \tag{1.3}$$

(d) The general split feasibility problem (GSFP) is formulated as the problem of finding a point x^* with the property:

$$x^* \in \bigcap_{i=1}^{\infty} C_i, \ Ax^* \in \bigcap_{i=1}^{\infty} Q_i. \tag{1.4}$$

There is a considerable investigation on CFP in view of its applications in various disciplines such as image restoration, computer tomograph, and radiation therapy treatment planning [1]. The split feasibility problem SFP in the setting of finite-dimensional Hilbert spaces was first introduced by Censor and Elfving [2] for modelling inverse problems which arise from phase retrievals and in medical image reconstruction [3]. Since then, a lot of work has been done on finding a solution of SFP and MSSFP; see, for example, [2–17].

In 2010, Xu [13] considered the SFP in the setting of infinite-dimensional Hilbert spaces and studied some algorithms and its convergence. In particular, he applied Mann's algorithm to the SFP and proposed an algorithm which is proved to be weakly convergent to a solution of the SFP. He also established the strong convergence result, which shows that the minimum-norm solution can be obtained.

In 2011, Wang and Xu [14] proposed the following cyclic algorithm to solve MSSFP:

$$x_{n+1} = P_{C[n]}(x_n + \gamma A^*(P_{Q[n]} - I)Ax_n), \tag{1.5}$$

where $[n] := n(modp)$ (mod function take values in $\{1, 2, \cdots, p\}$, and $\gamma \in (0, \frac{2}{||A||^2})$. They shown that the sequence $\{x_n\}$ converged weakly to a solution of MSSFP provided the solution exists.

To study strong convergence to a solution of MSSFP, in 2013, Eslamian and Latif [15] proposed the following algorithm to solve GSFP:

$$x_{n+1} = \alpha_n x_n + \beta_n f(x_n) + \sum_{i=1}^{\infty} \gamma_{n,i} P_{C_i}(I - \lambda_{n,i} A^*(I - P_{Q_i})A)x_n, \tag{1.6}$$

where $\alpha_n + \beta_n + \sum\limits_{i=1}^{\infty} \gamma_{n,i} = 1$. Under suitable conditions, the sequence $\{x_n\}$ converged strongly to a solution of GSFP.

In 2013, He and Zhao [16] introduced the following relaxed CQ algorithm such that the strong convergence was guaranteed in infinite-dimensional Hilbert spaces:

$$x_{n+1} = P_{C_n}(\alpha_n u + (1 - \alpha_n)(x_n - \tau_n \nabla f_n(x_n))), \tag{1.7}$$

where $f_n(x) = \frac{1}{2}||(I - P_{Q_n})Ax||^2$ and $\nabla f_n(x) = A^*(I - P_{Q_n})Ax$.