Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2015,**35B**(6):1325–1338
Mathematica Acta Scientia 数学物理学报
http://actams.wipm.ac.cn

ASYMPTOTIC STABILITY OF TRAVELING WAVES FOR A DISSIPATIVE NONLINEAR EVOLUTION SYSTEM*

Mina JIANG (蒋咪娜)
The Hubei Key Laboratory of Mathematical Physics, School of Mathematics and Statistics, Central China Normal University, Wuhan 430079, China
E-mail: jmn3911@mail.ccnu.edu.cn

Jianlin XIANG (向建林)†
Department of Mathematics, School of Science, Wuhan University of Technology, Wuhan 430079, China
E-mail: jlxiang2@163.com

Abstract This paper is concerned with the existence and the nonlinear asymptotic stability of traveling wave solutions to the Cauchy problem for a system of dissipative evolution equations

$$\begin{cases} \xi_t = -\theta_x + \beta\xi_{xx}, \\ \theta_t = \nu\xi_x + (\xi\theta)_x + \alpha\theta_{xx}, \end{cases}$$

with initial data and end states

$$(\xi, \theta)(x, 0) = (\xi_0, \theta_0)(x) \to (\xi_\pm, \theta_\pm) \quad \text{as} \quad x \to \pm\infty.$$

We obtain the existence of traveling wave solutions by phase plane analysis and show the asymptotic nonlinear stability of traveling wave solutions without restrictions on the coefficients α and ν by the method of energy estimates.

Key words dissipative evolution equations; traveling wave solutions; nonlinear stability; energy estimates

2010 MR Subject Classification 35K45; 35C07; 35B40

1 Introduction and Main Results

In physical and mechanical fields, many phenomena can be modeled by the systems of the nonlinear interaction between ellipticity and dissipation. Lorenz derived his famous equations

*Received September 3, 2014; revised November 24, 2014. Jiang's research was supported by the Natural Science Foundation of China (11001095), the Ph.D. specialized grant of the Ministry of Education of China (20100144110001) and the Special Fund for Basic Scientific Research of Central Colleges (CCNU12C01001). Xiang's research was supported by the Fundamental Research Funds for the Central Universities (2015IA009) and the Natural Science Foundation of China (61573012).

†Corresponding author: Jianlin XIANG.

in [1] from Rayleigh-Benard equations [2]. But the Rayleigh-Benard problem is a system of two highly nonlinear partial differential equations with three independent variables, except for the linearized system, it is difficult to analyze those equations in any way besides the numerical computations. Therefor it is useful to construct a manageable partial differential equation and its reduced system. Hsieh [3] proposed the following alternative system to try to yield more and new insight to Rayleigh-Benard equations

$$\begin{cases} \xi_t = -(\sigma-\beta)\xi - \sigma\theta_x + \beta\xi_{xx}, \\ \theta_t = -(1-\alpha)\theta + \nu\xi_x + (\xi\theta)_x + \alpha\theta_{xx}, \end{cases} \tag{1.1}$$

where $(t,x) \in [0,+\infty) \times R$, σ, α, β and ν are all positive constants satisfying the relation $\beta < \sigma$ and $\alpha < 1$. $\xi(x,t)$ and $\theta(x,t)$ denote a stream function and a temperature function respectively.

As a preliminary work, Jian and Chen [4] first established the global existence of solutions of the system (1.1) with the initial condition $(\xi_0, \theta_0) \in H^1(R,R^2) \cap L^1(R,R^2)$, and the optimal decay rate and optimal decay order were obtained by Wang [5]. Hsiao and Jian [6] obtained the global existence of classical solutions for the initial boundary value problem of the system (1.1) with initial condition $(\xi_0, \theta_0) \in C^{2,\delta}([0,1]) * C^{2,\delta}([0,1])$ $(0 < \delta < 1)$ and periodic boundary condition

$$(\xi_0, \theta_0)(0) = (\xi_0, \theta_0)(1), \quad \big((\xi)_x, (\theta)_x\big)(0,t) = \big((\xi)_x, (\theta)_x\big)(1,t), \quad 0 \le t \le T.$$

Moreover, Tang and Zhao [7] studied the following modified system

$$\begin{cases} \xi_t = -(\sigma-\beta)\xi - \sigma\theta_x + \beta\xi_{xx}, \\ \theta_t = -(1-\alpha)\theta + \nu\xi_x + 2\xi\theta_x + \alpha\theta_{xx}. \end{cases} \tag{1.2}$$

When the initial data $(\xi_0, \theta_0) \in L^2(R,R^2)$, they established the global existence, nonlinear stability and optimal decay rate of the solution to (1.2) with suitable restrictions on coefficients σ, α, β and ν. Furthermore, if $(\xi_0, \theta_0) \in L^1(R,R^2)$, they obtained the optimal decay rates of solutions for system (1.2). Zhu and Wang [8] extended the above results to more general case in which initial data satisfy

$$(\xi_0(x), \theta_0(x)) \to (\xi_\pm, \theta_\pm), \quad \text{as} \quad x \to \pm\infty,$$

where $(\xi_+ - \xi_-, \theta_+ - \theta_-) \neq (0,0)$. Duan and Zhu [9] investigated the asymptotics of diffusion wave toward the solution of system (1.2). Zhu et al. [10] obtained the global existence, nonlinear stability and decay rates of the solution to the diffusion wave for system (1.2). The optimal convergence rates of solutions to diffusion wave for the Cauchy problem of (1.1) and (1.2) were derived in [11] and [12], respectively. Hsieh [3] briefly discussed the linear stability of possible periodic traveling wave solution to (1.2) in cases of $0 \le \alpha \le 1$, $\nu > \alpha$ as well as $\sigma > 0, \beta > 0$. But there has not been any rigorous results of traveling wave solutions to (1.1) or (1.2). The aim of this paper is to study the existence and the nonlinear stability of traveling wave solutions to the system (1.1). Considering that (1.1) is a system of second-order parabolic equations, it is very challenging to consider the traveling wave solutions due to the high dimensionally of the wave system. In this paper, we shall consider the following system

$$\begin{cases} \xi_t = -\theta_x + \beta\xi_{xx}, \\ \theta_t = \nu\xi_x + (\xi\theta)_x + \alpha\theta_{xx}, \end{cases} \tag{1.3}$$