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STABILITY OF SOME POSITIVE LINEAR OPERATORS ON COMPACT DISK*

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Abstract Recently, Popa and Raşa [27, 28] have shown the (in)stability of some classical operators defined on $[0, 1]$ and found best constant when the positive linear operators are stable in the sense of Hyers-Ulam. In this paper we show Hyers-Ulam (in)stability of complex Bernstein-Schurer operators, complex Kantrovich-Schurer operators and Lorentz operators on compact disk. In the case when the operator is stable in the sense of Hyers and Ulam, we find the infimum of Hyers-Ulam stability constants for respective operators.

Key words Hyers-Ulam stability; Bernstein-Schurer operators; Kantrovich-Schurer operators; Lorentz operators; stability constants

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1 Introduction

The equation of homomorphism is stable if every "approximate" solution can be approximated by a solution of this equation. The problem of stability of a functional equation was formulated by S.M. Ulam [35] in a conference at Wisconsin University, Madison in 1940: "Given a metric group $(G, ., \rho)$, a number $\varepsilon > 0$ and a mapping $f : G \to G$ which satisfies the inequality $\rho(f(xy), f(x)f(y)) < \varepsilon$ for all $x, y \in G$, does there exist a homomorphism a of G and a constant $k > 0$, depending only on G, such that $\rho(a(x), f(x)) \le k\varepsilon$ for all $x \in G$?" If the answer is affirmative the equation $a(xy) = a(x)a(y)$ of the homomorphism is called stable; see [10, 17]. The first answer to Ulam's problem was given by D.H. Hyers [16] in 1941 for the Cauchy functional equation in Banach spaces, more precisely he proved: "Let X, Y be Banach spaces, ε a non-negative number, $f : X \to Y$ a function satisfying $\|f(x+y)-f(x)-f(y)\| \le \varepsilon$ for all $x, y \in X$, then there exists a unique additive function with the property $\|f(x)-a(x)\| \le \varepsilon$ for all $x \in X$." Due to the question of Ulam and the result of Hyers this type of stability is called today Hyers-Ulam stability of functional equations. A similar problem was formulated and solved earlier by G. Pólya and G. Szegö in [25] for functions defined on the set of positive integers. After Hyers result a large amount of literature was devoted to study Hyers-Ulam stability for various equations. A new type of stability for functional equations was introduced by T. Aoki [2] and Th.M. Rassias [29] by replacing ε in the Hyers theorem with a function depending on x and y, such that the Cauchy difference can be unbounded. The results of Aoki and Rassias have been

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complemented later in the papers [12] and [7]. Moreover, a lot of useful recent information on that type of stability can be found in [6].

The Hyers-Ulam stability of linear operators was considered for the first time in the papers by Miura, Takahasi et al. (see [14, 15, 21]). Similar type of results are obtained in [34] for weighted composition operators on $C(X)$, where X is a compact Hausdorff space. A result on the stability of a linear composition operator of the second order was given by J. Brzdek and S.M. Jung in [9].

Recently, Popa and Raşa obtained [26] a result on Hyers-Ulam stability of the Bernstein-Schnabl operators using a new approach to the Fréchet functional equation, and in [27, 28], they have shown the (in)stability of some classical operators defined on $[0,1]$ and found the best constant for the positive linear operators in the sense of Hyers-Ulam. For other results on the Hyers-Ulam stability of functional equations one can refer to [22, 23].

Motivated by their work, in this paper, we show the (in)stability of some complex positive linear operators on compact disk in the sense of Hyers-Ulam. We find the infimum of the Hyers-Ulam stability constants for complex Bernstein-Schurer operators and complex Kantrovich-Schurer operators on compact disk. Further we show that Lorentz polynomials are not stable in the sense of Hyers-Ulam on a compact disk. Issues considered in this paper are strictly connected with the problems of stability of the equation of fixed point investigated in [30]. Also, some related results have been obtained in [4, 5, 24, 31, 32, 36] and [8].

2 The Hyers-Ulam Stability Property of Operators

In this section, we recall some basic definitions and results on Hyers-Ulam stability property which form the background of our main results.

Definition 2.1 (see [34]) Let A and B be normed spaces and T a mapping from A into B. We say that T has the Hyers-Ulam stability property (briefly, T is HU-stable) if there exists a constant K such that:

(i) for any $g \in T(A)$, $\varepsilon > 0$ and $f \in A$ with $\|Tf - g\| \leq \varepsilon$, there exists an $f_0 \in A$ such that $Tf_0 = g$ and $\|f - f_0\| \leq K\varepsilon$. The number K is called a HUS constant of T, and the infimum of all HUS constants of T is denoted by K_T. Generally, K_T is not a HUS constant of T (see [14] and [15]).

Let now T be a bounded linear operator with the kernel denoted by $N(T)$ and the range denoted by $R(T)$. Consider the one-to-one operator $\widetilde{T}$ from the quotient space $A/N(T)$ into B:

$$\widetilde{T}(f + N(T)) = Tf, \quad f \in A,$$

and the inverse operator $\widetilde{T}^{-1} : R(T) \to A/N(T)$.

Theorem 2.2 (see [34]) Let A and B be Banach spaces and $T : A \to B$ be a bounded linear operator. Then the following statements are equivalent:

(a) T is HU-stable;

(b) $R(T)$ is closed;

(c) $\widetilde{T}^{-1}$ is bounded.

Moreover, if one of the conditions (a), (b), (c) is satisfied, then $K_T = \|\widetilde{T}^{-1}\|$.