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Asynchronous code-division random access using convex optimization ☆

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ABSTRACT

Many applications in cellular systems and sensor networks involve a random subset of a large number of users asynchronously reporting activity to a base station. This paper examines the problem of multiuser detection (MUD) in random access channels for such applications. Traditional orthogonal signaling ignores the random nature of user activity in this problem and limits the total number of users to be on the order of the number of signal space dimensions. Contention-based schemes, on the other hand, suffer from delays caused by colliding transmissions and the hidden node problem. In contrast, this paper presents a novel pairing of an asynchronous non-orthogonal code-division random access scheme with a convex optimization-based MUD algorithm that overcomes the issues associated with orthogonal signaling and contention-based methods. Two key distinguishing features of the proposed MUD algorithm are that it does not require knowledge of the delay or channel state information of every user and it has polynomial-time computational complexity. The main analytical contribution of this paper is the relationship between the performance of the proposed MUD algorithm in the presence of arbitrary or random delays and two simple metrics of the set of user codewords. The study of these metrics is then focused on two specific sets of codewords, random binary codewords and specially constructed algebraic codewords, for asynchronous random access. The ensuing analysis confirms that the proposed scheme together with either of these two codeword sets significantly outperforms the orthogonal signaling-based random access in terms of the total number of users in the system.

1. Introduction

Many applications of wireless networks require servicing a large number of users that share limited communication resources. In particular, the term *random access* is commonly used to describe a setup where a random subset of users in the network communicate with a base station (BS) in an uncoordinated fashion [1]. In this paper, we study random access in large networks for the case when active users transmit single bits to the BS. This so-called "on–off" random access channel (RAC) [2] represents an abstraction that arises frequently in many wireless networks. In third-generation cellular systems, for example, control channels used for scheduling requests can be modeled as on–off RACs; in this case, users requesting permissions to send data to the BS can be thought of as transmitting 1's and inactive users can be thought of as transmitting 0's. Similarly, uplinks in wireless sensor networks deployed for target detection can also be modeled

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as on–off RACs; in this case, sensors that detect a target can be made to transmit 1's and sensors that have nothing to report can be thought of as transmitting 0's.[1]

The primary objective of the BS in on–off RACs is to reliably and efficiently carry out multiuser detection (MUD), which translates into recovery of the set of active users in our case. The two biggest impediments to this goal are that (i) random access tends to be asynchronous in nature, and (ii) it is quite difficult, if not impossible, for the BS to know the channel state information (CSI) of every user. Given a fixed number of temporal signal space dimensions N in the uplink, the system-design goal therefore is to simultaneously maximize the total number of users M in the network and the average number of active users k that the BS can reliably handle *without* requiring knowledge of the delays or CSIs of the individual users at the BS.

Traditional approaches to random access fall significantly short of this design objective. In random access methods based on orthogonal signaling, the N signal space dimensions are orthogonally spread among the M users in either time, frequency, or code [1]. While this establishes a dedicated, interference-free channel between each user and the BS, this approach ignores the random nature of user activity in RACs. Therefore, by its very structure, random access based on orthogonal signaling dictates the relationship $k \leq M \leq N$. On the other hand, contention-based random access schemes such as ALOHA and carrier sense multiple access (CSMA) do take advantage of the random user activity [3]. However, significant problems arise in these schemes when the average number of active users k and/or the total number of users M gets large [3]. In the case of ALOHA, collisions and retransmissions accumulate to significant delays as k becomes large. In the case of CSMA, the number of potential "hidden nodes" grows as M increases, resulting in unintended and unrecognized collisions in large networks.

Cellular systems, partly because of the aforementioned reasons, typically resort to the use of matched filter receivers on uplink control channels. Such receivers correspond to single-user detection (SUD) since they detect each user independently, treating the interference from other active users as noise. However, despite the effectiveness of these receivers in today's cellular systems, SUD schemes also have significant pitfalls. In particular, such schemes tend to have suboptimal performance since they do not carry out joint detection and they tend to be prone to the "near–far" effect [2].

In order to overcome the issues associated with orthogonal signaling, contention-based methods and SUD schemes, we present in this paper a novel code-division random access (CDRA) scheme that spreads the uplink communication resources in a non-orthogonal manner among the M users and leverages the random user activity to service significantly more total users than N. A key distinguishing feature of the proposed scheme is that it makes use of a convex optimization-based MUD algorithm that does not require knowledge of the delays or CSIs of the users at the BS. In addition, we present an efficient implementation of the proposed algorithm based on the *fast Fourier transform* (FFT) that ensures that its computational complexity at worst differs by a logarithmic factor from an *oracle-based* MUD algorithm that has perfect knowledge of the user delays. Our main analytical contribution is the relationship between the probability of error P_{err} of the proposed MUD algorithm in the presence of arbitrary or random delays and two metrics of the set of codewords assigned to the users. We then make use of these metrics to analyze two specific sets of codewords, random binary codewords and specially constructed (deterministic) algebraic codewords, for the proposed random access scheme. Specifically, we show that both these codewords allow our scheme to successfully manage an average number of active users that is almost linear in $N : k \lesssim N/(\tau \log(M\tau))$ for arbitrary delays and $k \lesssim N/\log(M\tau)$ for uniformly random delays, where τ denotes maximum delay in the network. More importantly, we show that the set of random codewords enable our scheme to service a number of total users that (ignoring τ) is super-polynomial in N, $M \lesssim \exp(O(N^{1/3}))$, while the set of deterministic codes, which facilitate efficient codeword construction and storage, enable it to service a number of total users that is polynomial in N, $M \lesssim N^t$ for any reasonably sized $t \geq 2$.[2]

It is useful at this point to also consider non-orthogonal code-division *multiple access* (CDMA), which – like our scheme – also spreads the uplink communication resources in a non-orthogonal manner among $M > N$ users [1]. However, despite similarities at the codeword-assignment level, there are significant differences between non-orthogonal CDMA and the work presented here. First, non-orthogonal CDMA is used for applications in which a fixed set of users continually communicate with the BS, whereas our scheme corresponds to a random subset of users in a large network communicating single bits to the BS. Second, MUD schemes for non-orthogonal CDMA require that the BS has knowledge of the individual user delays, whereas we assume – partly because of the random user activity – that user delays are unknown at the BS.

In terms of related prior work, Fletcher et al. [2] have also recently studied the problem of MUD in on–off RACs. However, the results in [2] – while similar in spirit to the ones in here – are limited by the facts that [2]: (i) assumes perfect synchronization among the M users, which is hard to guarantee in practical settings for large M; (ii) assumes that CSIs of the individual users are available to the BS in certain cases, which is difficult – if not impossible – to justify for the case of *fading* RACs; and (iii) only guarantees that the probability of error P_{err} at the BS goes to zero asymptotically in M, which does not shed light on the scaling of P_{err}. More recently, we have become aware of the independent and simultaneous work in [4,5] that

1 The focus of this paper is on servicing a large number of users that share limited communication resources in the uplink. Limiting ourselves to on–off RACs in this case helps us isolate the key issues associated with designing arbitrary RACs involving (multiple-bit) packet transmissions in large networks.

2 Recall the "*Big-O*" notation: $f(n) = O(g(n))$ (alternatively, $f(n) \lesssim g(n)$) if $\exists c_o > 0, n_o : \forall n \geq n_o, f(n) \leq c_o g(n)$.