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# STABILITY RESULTS OF RANDOM IMPULSIVE SEMILINEAR DIFFERENTIAL EQUATIONS*

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**Abstract** In this paper, we study the existence, uniqueness, continuous dependence, Ulam stabilities and exponential stability of random impulsive semilinear differential equations under sufficient condition. The results are obtained by using the contraction mapping principle. Finally an example is given to illustrate the applications of the abstract results.

**Key words** semilinear differential equations; random impulses; stability, Hyers-Ulam stability; Hyers-Ulam-Rassias stability; exponential stability; contraction principle

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## 1 Introduction

Impulsive differential equations are well known to model problems from many areas of science and engineering. There has been much research activity concerning the theory of impulsive differential equations see [1, 2]. The impulses may exists at deterministic or random points. There are lot of papers which investigate the properties of deterministic impulses see [1–4] and the references therein.

When the impulses exists at random points, then the solutions of the differential equations is a stochastic process. It is very different from deterministic impulsive differential equations and also it is different from stochastic differential equations. Thus the random impulsive equations gives more realistic than deterministic impulsive equations. There are few publications in this field, Wu and Meng first brought forward random impulsive ordinary differential equations and investigated boundedness of solutions to these models by Liapunov's direct function in [5]. Wu

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et al, studied some qualitative properties of random impulses in [6–9]. In [10], the author studied the existence and exponential stability for a random impulsive semilinear functional differential equations through the fixed point technique under non-uniqueness. The existence, uniqueness and stability results were discussed in [11] through Banach fixed point method for the system of differential equations with random impulsive effect. In [12–15] the author studied the existence results for the random impulsive neutral functional differential equations and differential inclusions with delays. In [16], the authors generalized the distribution of random impulses with the Erlang distribution.

The stabilities like continuous dependence, Hyers-Ulam stability, Hyers-Ulam-Rassias stability, exponential stability and asymptotic stability have attracted the attention of many mathematicians (see [17–24] and the references therein). In [21], the authors have given the Ulam's type stability and data dependence for fractional differential equations (FDEs). Wang et al. [22] studied stability of FDEs using fixed point theorem in a generalized complete metric space. In [24], Wang et al. studied Ulam's stability for the nonlinear impulsive FDEs. Moreover, Burton and Zhang [27], studied the existence and asymptotic stability through fixed point theory. Luo [28] studied the exponential stability and almost sure exponential stability in $p$-th mean of mild solutions of stochastic differential equation by means of contraction mapping principle.

Motivated by the above mentioned works, the main purpose of this paper is to study of random impulsive semilinear differential systems. We relaxed the Lipschitz condition on the impulsive term and under our assumption it is enough to be bounded. We extend the results of Hyers-Ulam stability and Hyers-Ulam-Rassias stability to fill the gab in abstract partial differential equation. We utilize the technique developed in [1, 2, 7, 21, 24–26, 28–31].

The paper will be organized as follows: In Section 2, we recall briefly the notations, definitions, preliminary facts, existence and uniqueness theorem which are used throughout this paper. In Section 3, we study the stability through continuous dependence on initial conditions of random impulsive semilinear differential systems. The Hyers Ulam stability and Hyers Ulam-Rassias stability of the solutions of differential systems is investigated in Section 4. We study the exponential stability of solution of random impulsive semilinear differential equations with delays in Section 5 by using contraction mapping principle. Finally in Section 6, an example is presented to show our results.

## 2 Preliminaries

Let $X$ be a real separable Hilbert space and $\Omega$ a nonempty set. Assume that $\tau_k$ is a random variable defined from $\Omega$ to $D_k \overset{\text{def.}}{=} (0, d_k)$ for $k = 1, 2, \cdots$, where $0 < d_k < +\infty$. Furthermore, assume that $\tau_k$ follow Erlang distribution, where $k = 1, 2, \cdots$ and let $\tau_i$ and $\tau_j$ are independent with each other as $i \neq j$ for $i, j = 1, 2, \cdots$. For the sake of simplicity, we denote $\Re_\tau = [\tau, +\infty), \Re^+ = [0, +\infty)$.

We consider semilinear differential equations with random impulses of the form

$$\begin{cases} x'(t) = Ax(t) + f(t, x_t), t \neq \xi_k, \quad t \geq t_0, \\ x(\xi_k) = b_k(\tau_k)x(\xi_k^-), \qquad k = 1, 2, \cdots, \\ x_{t_0} = \varphi, \end{cases} \tag{2.1}$$