Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2014,**34B**(4):1254–1270
Mathematica Acta Scientia
数学物理学报
http://actams.wipm.ac.cn

EXPONENTIAL STABILITY FOR NONLINEAR HYBRID STOCHASTIC PANTOGRAPH EQUATIONS AND NUMERICAL APPROXIMATION*

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Abstract The paper develops exponential stability of the analytic solution and convergence in probability of the numerical method for highly nonlinear hybrid stochastic pantograph equation. The classical linear growth condition is replaced by polynomial growth conditions, under which there exists a unique global solution and the solution is almost surely exponentially stable. On the basis of a series of lemmas, the paper establishes a new criterion on convergence in probability of the Euler-Maruyama approximate solution. The criterion is very general so that many highly nonlinear stochastic pantograph equations can obey these conditions. A highly nonlinear example is provided to illustrate the main theory.

Key words stochastic pantograph equation; hybrid system; polynomial growth conditions; exponential stability; convergence in probability

2010 MR Subject Classification 65C20; 65C30

1 Introduction

The hybrid systems driven by continuous-time Markovian chains have been used to model many practical systems where they may experience abrupt changes in their structure and parameters. Such systems combine a part of the state that takes values continuously and another part of the state that takes discrete values. Such hybrid systems have received an increasing attention [1–3]. Altans [4] suggested that hybrid systems would become a basic framework in posing and solving control-related issues in battle management command, control, and communications systems. An important class of hybrid systems is hybrid stochastic unbounded delay system

$$\mathrm{d}x(t) = f(x(t), x(qt), r(t))\mathrm{d}t + g(x(t), x(qt), r(t))\mathrm{d}w(t), \quad 0 < q < 1 \tag{1.1}$$

*Received May 28, 2012; revised August 15, 2013. The financial support from the National Natural Science Foundation of China (70871046, 71171091, 71191091) and Fundamental Research Funds for the Central Universities (2011QN167)

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take values in $\mathbb{R}^n$ while another part of the state $r(t)$ is a markov chain taking values in a finite space $S = \{1, 2, \cdots, N\}$. Such a hybrid stochastic pantograph system is also known as a stochastic pantograph system with Markovian switching.

Stochastic unbounded delay systems play an important role in a variety of application areas, including biology, epidemiology, mechanic, economics and finance. The systems provide powerful models, such as, infinite delay Kolmogorov-type systems in mathematic biology [5, 6], stochastic neural networks [7–9], stochastic pantograph equations in science and engineering. So such systems have received an increasing attention [10–15]. The pantograph equation which is a very special unbounded delay equation was used by Ockendon and Taylor [16] in 1971 to study how the electric current is collected by the pantograph of an electric locomotive, from where it gets its name. Baker and Buckwar [17] established the existence-and-uniqueness of the analytical solution for the linear stochastic pantograph equation. Appleby [18] investigated the growth and decay rates of the solutions for scalar stochastic pantograph equations with a linear drift which contains an unbounded delay term, a nonlinear diffusion term, which depends on the current state only. Meng et al. [19] investigated the existence and uniqueness and the pathwise estimation of the global solution for stochastic pantograph equation by using the Lyapunov function method and the exponential martingale inequality.

However, most of stochastic differential equations cannot be solved explicitly. Especially, explicit solutions can rarely be obtained for nonlinear hybrid stochastic pantograph equations, so numerical methods have recently received more and more attention [20, 21]. For example, Fan et al. [22] investigated the convergence of the semi-implicit Euler methods for stochastic pantograph equations under the Lipschitz condition and the linear growth condition. Li et al. [23] studied convergence in probability of the Euler approximation solution for stochastic pantograph equations with Markovian switching under weaker conditions. Fan et al. [24] investigated the numerical approximation for the linear stochastic pantograph equation by using the Razumikhin technique. Milošević et al. [25] studied a Taylor polynomial approximate solution for stochastic pantograph equation with Markovian switching under the linear growth condition.

Unfortunately, the existing results of convergence derived by these conditions are somewhat restrictive for the purpose of practical applications, because there are many stochastic differential equations that only satisfy the local Lipschitz condition. Recently, Mao et al. [26] developed convergence in probability of the numerical approximate solution of stochastic differential delay equation under the Khasminskii-type condition. Milošević [27] studied convergence in probability of the Euler-Maruyama approximate solution for neutral stochastic differential delay equation under the Khasminskii-type condition. Zhou et al. [28] studied convergence in probability of the Euler-Maruyama approximate solution of nonlinear neutral stochastic functional differential equation.

To the best of the authors' knowledge, there is so far little numerical theory on highly nonlinear hybrid stochastic pantograph equation yet. The paper shall develop a new criterion on convergence of the Euler-Maruyama method for highly nonlinear hybrid stochastic pantograph equation. The classical linear growth condition is replaced by polynomial growth conditions, under which there exists a unique global solution and the solution is almost surely exponentially stable. On basis of a series of lemmas, we show that the Euler-Maruyama approximate solution