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WELL-POSEDNESS OF A COMPRESSIBLE GAS-LIQUID MODEL FOR DEEPWATER OIL WELL OPERATIONS*

Helmer A. FRIIS†
International Research Institute of Stavanger (IRIS), Prof. Olav Hanssensvei 15, NO-4068 Stavanger, Norway
E-mail: HelmerAndre.Friis@iris.no
Steinar EVJE
University of Stavanger, NO-4068 Stavanger, Norway
E-mail: steinar.evje@uis.no

Abstract The main purpose of this paper is two-fold: (i) to generalize an existence result for a compressible gas-liquid model with a friction term recently published by Friis and Evje [SIAM J. Appl. Math., 71 (2011), pp. 2014–2047]; (ii) to derive a uniqueness result for the same model. A main ingredient in the existence part is the observation that we can consider weaker assumptions on the initial liquid and gas mass, and still obtain an existence result. Compared to the above mentioned work, we rely on a more refined application of the estimates provided by the basic energy estimate. Concerning the uniqueness result, we borrow ideas from Fang and Zhang [Nonlinear Anal. TMA, 58 (2004), pp. 719–731] and derive a stability result under appropriate constraints on parameters that determine rate of decay toward zero at the boundary for gas and liquid masses, and growth rate of masses associated with the friction term and viscous coefficient.

Key words two-phase flow; well model; gas-kick; weak solutions; Lagrangian coordinates; free boundary problem; friction term; uniqueness

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1 Introduction

This work is devoted to a study of a transient gas-liquid two-phase model which, in Lagrangian variables, takes the following form:

$$\begin{aligned} &\partial_t n + (n\zeta)\partial_x u = 0, \\ &\partial_t \zeta + \zeta^2 \partial_x u = 0, \\ &\partial_t u + \partial_x p(n,\zeta) = -f\zeta^\beta u|u| + \partial_x (E(n,\zeta)\partial_x u), \quad x \in (0,1) \end{aligned} \tag{1.1}$$

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†Corresponding author: Helmer A. FRIIS.

with constants $f, \beta > 0$. Here n is the gas mass, ζ the total mass (sum of gas and liquid mass), whereas u is the common fluid velocity. The pressure law, when liquid is assumed to be incompressible (ρ_l=const) and gas is treated as an ideal gas, takes the form

$$p(n,\zeta) = \left(\frac{n}{\rho_l - [\zeta - n]}\right)^{\gamma}, \qquad \gamma > 1. \tag{1.2}$$

The first term on the right hand side of the momentum equation represents wall friction where the parameter $\beta > 0$ describes the mass decay rate toward zero, whereas the second term takes into account other viscous effects and is characterized by the coefficient

$$E(n,\zeta) := \left(\frac{\zeta}{(\rho_l - [\zeta - n])}\right)^{\theta+1}, \qquad 0 < \theta < 1/2. \tag{1.3}$$

Moreover, boundary conditions are given by

$$n(0,t) = \zeta(0,t) = 0, \qquad n(1,t) = \zeta(1,t) = 0, \tag{1.4}$$

whereas initial data are

$$n(x,0) = n_0(x), \quad \zeta(x,0) = \zeta_0(x), \quad u(x,0) = u_0(x), \qquad x \in (0,1). \tag{1.5}$$

This model problem represents a natural continuation of the work [15] where an existence result was established for a similar model with inclusion of external forces like gravity and friction. In turn, this work builds upon the works [9, 10], see also [30, 31] for related interesting results.

In the recent work [16] we considered the model problem (1.1)–(1.5) for the case when the gas and liquid mass vanish at the boundary. A main concern in that work was inclusion and analysis of effects related to wall friction. The friction term is important for realistic predictions of the pressure profile along wellbore, which is crucial for a good understanding of mechanisms for safe handling of a gas-kick.

In particular, an existence result was obtained under appropriate assumptions on the parameters γ, θ, and β appearing in (1.1), (1.2), and (1.3). The heart of the matter in the analysis is the use of an appropriate variable transformation which allows writing the two-phase model in a form which naturally opens up for exploiting single-phase techniques [18, 19, 21–23, 25, 26, 29, 32–34]. It turns out that we naturally can reformulate the initial boundary value (IBV) problem (1.1)–(1.5) described in terms of the variables (n, ζ, u) into a corresponding IBV problem described in terms of the variables (c, Q, u) where $c = n/\zeta$ and $Q(c,\zeta) = \zeta/(\rho_l - [1-c]\zeta)$. The model then takes the form

$$\begin{aligned} \partial_t c &= 0, \\ \partial_t Q + \rho_l Q^2 u_x &= 0, \\ \partial_t u + \partial_x p(cQ) &= -h(c,Q)u|u| + \partial_x(E(Q)\partial_x u) \end{aligned}$$

with

$$p(cQ) = (cQ)^{\gamma}, \qquad h(c,Q) = f\rho_l^{\beta}\left(\frac{Q}{1+(1-c)Q}\right)^{\beta}, \qquad E(Q) = Q^{\theta+1}.$$

This reformulated version allows us to explore the role played by the frictional term. A main observation was that we could derive the necessary estimates by relying on assumption (2.15)

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