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PARAMETER IDENTIFICATION IN FRACTIONAL DIFFERENTIAL EQUATIONS*

Jing LI (李景)
School of Mathematics and Computational Science, Changsha University of Science and Technology, Changsha 410004, China
Institute of Applied Physics and Computational Mathematics, Beijing 100088, China
E-mail: *lijingnew@126.com*

Boling GUO (郭柏灵)
Institute of Applied Physics and Computational Mathematics, Beijing 100088, China
E-mail: *gbl@iapcm.ac.cn*

Abstract This article investigates the fractional derivative order identification, the coefficient identification, and the source identification in the fractional diffusion problems. If $1<\alpha<2$, we prove the unique determination of the fractional derivative order and the diffusion coefficient $p(x)$ by $\int_0^t u(0,s)\mathrm{d}s, 0<t<T$ for one-dimensional fractional diffusion-wave equations. Besides, if $0<\alpha<1$, we show the unique determination of the source term $f(x,y)$ by $U(0,0,t), 0<t<T$ for two-dimensional fractional diffusion equations. Here, α denotes the fractional derivative order over t.

Key words Fractional differential equation; inverse problems; parameter identification

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1 Introduction

Over the past two decades, because the strong relationship between the fractional derivative operator with the fractional Brownian motion, the continuous time random walk method [1, 2], the Lévi stable distributions, and the generalized central limit theorem, fractional equations have come of age as a complementary tool for the modeling of many anomalous phenomena in nature and in the theory of complex systems (such as the charge transport in amorphous semiconductors, the spread of contaminants in underground water, the relaxation in polymer systems, and tracers dynamics both in polymer networks and in arrays of convection rolls). The readers may refer to [1–8] and references therein. We can model them as the following fractional equation:

$$\partial_{0+}^{\alpha} u(x,t)=\operatorname{div}(p(x)\nabla u)+f(x,t),\ x\in\Omega,\ t\in(0,T),\quad 0<\alpha<2, \tag{1.1}$$

where Ω is a bounded domain in $R^{\mathcal{N}}$ with a piecewise smooth boundary $\partial\Omega$, $T>0$ is a fixed value, $p(x)\in C^2(\Omega)>0$ is the diffusion coefficient, and $f(x,t)$ is the source term. Here,

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$\partial_{0+}^{\alpha} u(x,t)$ denotes the Caputo fractional derivative acting on the time variable, which is defined by

$$\partial_{a+}^{\alpha} u(x,t) = \begin{cases} \dfrac{1}{\Gamma(n-\alpha)} \displaystyle\int_a^t (t-\tau)^{n-\alpha-1} \frac{\mathrm{d}^n}{\mathrm{d}\tau^n} u(x,\tau)\mathrm{d}\tau, \; n-1<\alpha<n, \; n \in \mathcal{N}, \\ \dfrac{\mathrm{d}^n}{\mathrm{d}t^n} u(x,t), \qquad \alpha = n \in \mathcal{N}, \end{cases} \tag{1.2}$$

where Γ is the Gamma function. For $0<\alpha<1$, (1.1) is called a fractional diffusion equation, while (1.1) is called a fractional diffusion-wave equation in the case of $1<\alpha<2$.

In 1986, fractional initial/boundary value problems were firstly considered by Nigmatullin [9]. From then on, there have been more and more articles on analytical solutions and numerical solutions of fractional initial/boundary value problems. Especially, using integral transform method, composition method, and operational method, Klias [10] and Podlubny [11] constructed explicit solutions of fractional initial/boundary value problems. Guo et al [12] outlined numerical solutions of fractional Schrödinger equations, fractional Ginzburg-Landau equations, fractional QG equations, and so on. Sakamoto and Yamamoto [13] proved the unique existence of weak solution applying the eigenfunction expansions, and determined t-varying factor in the source by observation at one point over $(0,T)$ for $0<\alpha<2$.

However, to the best of the authors' knowledge, there is a little work on inverse problems for fractional differential equations. By Gel'fand-Levitan theory, in one-dimensional environment, Cheng et al [14] put forward the unique determination of $\alpha \in (0,1)$ and $p(x), 0<x<l$ by $u(0,t), 0<t<T$ if $f(x,t)=0$ in (1.1). In [15], Liu et al considered a backward problem in time for a one-dimensional time-fractional diffusion equation. Murio and Mejía [16] introduced a regularization technique for the approximate reconstruction of spatial and time varying source terms using the observed solutions of the forward time fractional diffusion problems on a discrete set of points. Nakagawa et al [17] proposed that the solution can be uniquely determined by data in any small subdomain over time interval. Tuan [18] presented that by taking suitable initial distributions only finitely many measurements on the boundary are required to recover uniquely the diffusion coefficient of a one-dimensional fractional diffusion equation. Zhang and Xu [19] outlined that if $p(x)=1$ and $f(x,t)$ is only dependant on x, the unknown source term $f(x)$ can also be uniquely determined by $u(0,t), 0<t<T$. Additionally, they constructed a regularization scheme to obtain the regularized solution. We remark that α involved in all the above articles was assumed to be $0<\alpha<1$, and most of the above fractional inverse problems are involved in one-dimensional spaces. The super-diffusion processes $(1<\alpha<2)$ and the fractional inverse problems in higher-dimensional spaces are still studied less frequently. Therefore, throughout this article, we focus on the next two problems:

- Problem (1): For $1<\alpha<2, \Omega=(0,l)$, and $f(x,t)=0$ in (1.1), consider the following problem:

$$\begin{cases} \partial_{0+}^{\alpha} u(x,t) = (p(x)u_x)_x, \; 0<x<l, \; 0<t<T, \\ u_x(0,t) = u_x(l,t) = 0, \quad 0<t<T, \\ u(x,0) = \delta(x), \qquad 0<x<l, \end{cases} \tag{1.3}$$

where $\delta(x)$ is the Dirac delta function. Determine $p(x)>0$ and α by $\int_0^t u(0,s)\mathrm{d}s, 0<t<T$.