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THE VALUE DISTRIBUTION AND UNIQUENESS OF ONE CERTAIN TYPE OF DIFFERENTIAL-DIFFERENCE POLYNOMIALS*

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Abstract In this article, we investigate the distribution of the zeros and uniqueness of differential-difference polynomials

$$G(z) = (f^n(f^m(z) - 1)\prod_{j=1}^{d} f(z + c_j)^{v_j})^{(k)} - \alpha(z),$$

$$H(z) = (f^n(f(z) - 1)^m \prod_{j=1}^{d} f(z + c_j)^{v_j})^{(k)} - \alpha(z),$$

where f is transcendental entire function of finite order, $c_j (j = 1, 2, \cdots, d)$, n, m, d, and $v_j (j = 1, 2, \cdots, d)$ are integers, and obtain some theorems, which extended and improved many previous results.

Key words Meromorphic; uniqueness; value distribution; differential-difference

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1 Introduction and Main Results

In this article, we assume that reader is familiar with the standard notations and results such as the proximity functions $m(r, f)$, counting function $N(r, f)$, characteristic function $T(r, f)$, the first and second main theorems, lemma on the logarithmic derivatives of Nevanlinna theory, see [1–3].

Let p be a positive integer and $a \in \mathbf{C}\bigcup\{\infty\}$, then we denote by $N_p(r, \frac{1}{f-a})$ the counting function of the zeros of $f - a$, where an m-fold zero is counted m times if $m \le p$ and p times if $m > p$. We also need the following definition: Let f be a nonconstant meromorphic function, we

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define difference operators as $\Delta_\eta f(z)=f(z+\eta)-f(z), \Delta_\eta^n f(z)=\Delta_\eta^{n-1}(\Delta_\eta f(z))$, where η is a nonzero complex number and $n\geq 2$ is a positive integer. If $\eta=1$, we denote $\Delta_\eta f(z)=\Delta f(z)$.

The difference logarithmic derivative lemma, given by Chiang and Feng [4], Halburd and Korhonen [5], plays an important part in considering the difference analogues of Nevanlinna theory. With the development of difference analogue of Nevanlinna theory, many authors paid their attention to the zero distribution of difference polynomials [6–16]. Liu and Yang [9] also considered the zeros of $f^n(z)f(z+c)-p(z)$ and $f^n\triangle_c f$, where $p(z)$ is a nonzero polynomial, and obtain the following theorem:

Theorem A Let f be a transcendental entire function of finite order and $p(z)$ be a nonzero polynomial. If $n\geq 2$, then $f^n(z)f(z+c)-p(z)$ has infinitely many zeros. If f is not a periodic function with period c and $n\geq 2$, then, $\triangle_c f=f(z+c)-f(z)$ has infinitely many zeros.

In 2010, Zhang [17] considered zeros of one certain type of difference polynomials and obtained the following theorem.

Theorem B Let f be transcendental entire function of finite order, $\alpha(z)\not\equiv 0$ be a small function with respect to $f(z)$, $c_j(j=1,2,\cdots,d)$, c be nonzero complex constant, and n be an integer. If $n\geq 2$, then, $f^n(z)(f(z)-1)f(z+c)-\alpha(z)$ has infinitely many zeros.

In 2012, Chen and Chen [18] considered zeros of one certain type of difference polynomials and obtained the following theorem.

Theorem C Let f be transcendental entire function of finite order, $\alpha(z)\not\equiv 0$ be a small function with respect to $f(z)$, $c_j(j=1,2,\cdots,d)$, n,m,d, and $v_j(j=1,2,\cdots,d)$ be integers. If $n\geq 2$, then, $f^n(f^m(z)-1)\prod\limits_{j=1}^{d}f(z+c_j)^{v_j}-\alpha(z)$ has infinitely many zeros.

Theorem D Let f and g be two transcendental entire functions of finite order, $\alpha(z)\not\equiv 0$ be a common small function with respect to f and g, c be nonzero finite complex numbers. If $n\geq m+8\sigma$, n,m,d, and $v_j(j=1,2,\cdots,d)$ are integers, and $f^n(f^m(z)-1)\prod\limits_{j=1}^{d}f(z+c_j)^{v_j}$ and $g^n(g^m(z)-1)\prod\limits_{j=1}^{d}g(z+c_j)^{v_j}$ share $\alpha(z)$ CM, then, $f=tg$, where $t^m=t^{n+\sigma}=1$.

In this article, we investigate the following difference polynomial:

$$(f^n(f^m(z)-1)\prod_{j=1}^{d}f(z+c_j)^{v_j})^{(k)} \quad\text{and}\quad (f^n(f(z)-1)^m\prod_{j=1}^{d}f(z+c_j)^{v_j})^{(k)},$$

where f is transcendental entire function of finite order, $c_j(j=1,2,\cdots,d)$, n,m,d, and $v_j(j=1,2,\cdots,d)$ are nonnegative integers, and $\sigma=\sum\limits_{j=1}^{d}v_j$.

Theorem 1 Let f be transcendental entire function of finite order, $\alpha(z)\not\equiv 0$ be a small function with respect to f, $c_j(j=1,2,\cdots,d)$ be distinct finite complex numbers, and n,m,d, and $v_j(j=1,2,\cdots,d)$ be nonnegative integers. If $n\geq k+2$, then, the differential-difference polynomial $(f^n(f^m(z)-1)\prod\limits_{j=1}^{d}f(z+c_j)^{v_j})^{(k)}-\alpha(z)$ has infinitely many zeros.

Remark 1 If $k=0$, we can easily get Theorem C.

Theorem 2 Let f be transcendental entire function of finite order, $\alpha(z)\not\equiv 0$ be a small function with respect to f, $c_j(j=1,2,\cdots,d)$ be distinct finite complex numbers, and n,m,d, and $v_j(j=1,2,\cdots,d)$ are nonnegative integers. If one of the following conditions holds: