Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2014,**34B**(3):941–959
Acta Mathematica Scientia
数学物理学报
http://actams.wipm.ac.cn

BOUNDED TRAVELING WAVE SOLUTIONS OF VARIANT BOUSSINESQ EQUATION WITH A DISSIPATION TERM AND DISSIPATION EFFECT*

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Abstract This article studies bounded traveling wave solutions of variant Boussinesq equation with a dissipation term and dissipation effect on them. Firstly, we make qualitative analysis to the bounded traveling wave solutions for the above equation by the theory and method of planar dynamical systems, and obtain their existent conditions, number, and general shape. Secondly, we investigate the dissipation effect on the shape evolution of bounded traveling wave solutions. We find out a critical value r^* which can characterize the scale of dissipation effect, and prove that the bounded traveling wave solutions appear as kink profile waves if $|r| \geq r^*$; while they appear as damped oscillatory waves if $|r| < r^*$. We also obtain kink profile solitary wave solutions with and without dissipation effect. On the basis of the above discussion, we sensibly design the structure of the approximate damped oscillatory solutions according to the orbits evolution relation corresponding to the component $u(\xi)$ in the global phase portraits, and then obtain the approximate solutions $(\widetilde{u}(\xi), \widetilde{H}(\xi))$. Furthermore, by using homogenization principle, we give their error estimates by establishing the integral equation which reflects the relation between exact and approximate solutions. Finally, we discuss the dissipation effect on the amplitude, frequency, and energy decay of the bounded traveling wave solutions.

Key words Variant Boussinesq equation with dissipation term; shape analysis; bounded traveling wave solution; error estimate; dissipation effect

2010 MR Subject Classification 35G20; 35Q35; 35Q51

*Received May 9, 2013; revised July 17, 2013. The first author is supported by National Natural Science Foundation of China (11071164), Innovation Program of Shanghai Municipal Education Commission (13ZZ118), and Shanghai Leading Academic Discipline Project (XTKX2012).

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1 Introduction

Variant Boussinesq equation

$$\begin{cases} u_t + H_x + uu_x = 0, \\ H_t + (Hu)_x + u_{xxx} = 0 \end{cases} \tag{1.1}$$

is the mathematical model which can describe the propagation of surface long wave towards two directions in a certain deep trough. It can be converted from the classical Boussinesq equation [1–3]

$$\begin{cases} u_t + \rho_x + uu_x = 0, \\ \rho_t + u_x + u_{xxx} + (\rho u)_x = 0 \end{cases} \tag{1.2}$$

by the transformation $H(x,t) = 1 + \rho(x,t)$ [3]. In Eq.(1.1), $u(x,t)$ is the wave speed, while $H(x,t)$ is the height of free wave surface for fluid in the trough [1].

Many researchers studied Eq.(1.1). Sach [3] studied the Painlevé property, rational solutions and its equivalent relation with AKNS system; Kaup [4] and Ablowite [5] presented its inverse scattering transformation solutions; Kapershmidt [6] discussed the symmetry and conservation law of Eq.(1.1); Wang [7] and Zhang [8] obtained solitary wave solutions and multi-solitary wave solutions of Eq.(1.1) by using homogeneous balance method and generalized homogeneous balance method; Fan [9] applied algebraic method and found out two solitary wave solutions and a Jacobin function periodic solution of Eq.(1.1); Yuan [10] employed bifurcation theory in dynamical system to study Eq.(1.1), classified its solution orbits, and presented its corresponding integral expressions of bell profile solutions and kink profile solutions.

Recently, elliptic function traveling wave solutions and trigonometric function traveling wave solutions of Eq.(1.1) have been studied in many references (see [11–13]). The method of variation iteration also is applied in [14] to solve Eq.(1.1), and the solution in the form of infinite power series under proper initial value was obtained, whose approximation was also verified.

Because wave comes across the damping in the movement, dissipation would rise. G.B. Whitham [1] pointed out that one of basic problems which need to be concerned for nonlinear evolution equations is how dissipation affects nonlinear system. Therefore, this article considers the following variant Boussinesq equation with the dissipation term

$$\begin{cases} u_t + H_x + uu_x = 0, \\ H_t + (Hu)_x - ru_{xx} + u_{xxx} = 0. \end{cases} \quad r > 0 \tag{1.3}$$

Obviously, it is meaningful. r is the dissipation coefficient in Eq.(1.3). Eq.(1.3) becomes Eq.(1.1) if $r = 0$. We will focus on the shape evolution of traveling wave solutions for Eq.(1.3) which was accompanied by a dissipation term ($r > 0$), and how to solve it; meanwhile, we will point out the dissipation effect on this system. As the dissipation effect varies, how the shape of the bounded traveling wave solutions for Eq.(1.3) evolve? In this article, we will prove that the bounded traveling wave solutions appear as kink profile solitary waves if dissipation effect is large and they appear as damped oscillatory waves if dissipation effect is small. We will give the critical value which can characterize the scale of dissipation effect. More importantly, we will discuss how to obtain the approximate damped oscillatory solutions of Eq.(1.3) if r is small and the problem of presenting their error estimates (It is very important to give the error