Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2013,**33B**(1):84–106
Mathematica Acta Scientia 数学物理学报
http://actams.wipm.ac.cn

ELASTIC MEMBRANE EQUATION WITH MEMORY TERM AND NONLINEAR BOUNDARY DAMPING: GLOBAL EXISTENCE, DECAY AND BLOWUP OF THE SOLUTION*

Abderrahmane ZARAÏ
Depatment of Mathematics and Informatic, Cheikh El Arbi Tébessi University, 12002 Tébessa, Algeria
E-mail: zaraiabdoo@yahoo.fr
Nasser-eddine TATAR
Depatment of Mathematics and Statistics, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia
E-mail: tatarn@kfupm.edu.sa
Salem ABDELMALEK
Depatment of Mathematics, College of Sciences, Yanbu, Taibah University, Saudi Arabia
Depatment of Mathematics and Informatic, Cheikh El Arbi Tébessi University, 12002 Tébessa, Algeria
E-mail: sallllm@gmail.com

Abstract In this paper we consider the Elastic membrane equation with memory term and nonlinear boundary damping. Under some appropriate assumptions on the relaxation function h and with certain initial data, the global existence of solutions and a general decay for the energy are established using the multiplier technique. Also, we show that a nonlinear source of polynomial type is able to force solutions to blow up in finite time even in presence of a nonlinear damping.

Key words elastic membrane equation; global existence; boundary damping; boundary source; general decay; blowup

2010 MR Subject Classification 35L05; 35L20; 35B40; 45K05

1 Introduction

The objective of this work is to study the following initial-boundary value problem

$$\begin{cases} u_{tt} - M(t)\Delta u + \displaystyle\int_0^t h\,(t-s)\,\Delta u \mathrm{d}s = 0 & \text{in } \Omega \times (0,+\infty)\,, \\ u\,(x,0) = u_0\,(x) \ \text{ and } u_t\,(x,0) = u_1\,(x) & \text{in } \bar{\Omega}, \\ u = 0 & \text{on } \Gamma_1 \times [0,+\infty)\,, \\ M(t)\dfrac{\partial u}{\partial \nu} - \displaystyle\int_0^t h\,(t-s)\,\frac{\partial u}{\partial \nu}\mathrm{d}s + \alpha\,|u_t|^{m-2}\,u_t = |u|^{p-2}\,u & \text{on } \Gamma_0 \times [0,+\infty)\,. \end{cases} \tag{1.1}$$

*Received October 12, 2011; revised June 17, 2012.

where Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$ such that $\partial\Omega = \Gamma_0 \cup \Gamma_1$, $\bar{\Gamma}_0 \cap \bar{\Gamma}_1 = \emptyset$ and Γ_0, Γ_1 have positive measure $\lambda_{n-1}(\Gamma_i)$, $i = 0, 1$, ν denotes the unit outer normal vector pointing toward the exterior of Ω and $M(t) = \xi_0 + \xi_1 \|\nabla u(t)\|_2^2 + \sigma(\nabla u(t), \nabla u_t(t))$, where u is the plate transverse displacement, x is the spatial coordinate in the direction of the fluid flow, and t is time. The viscoelastic structural damping terms are denoted by σ, ξ_1 is the nonlinear stiffness of the membrane, and ξ_0 is an in-plane tensile load. All quantities are physically non-dimensionalized and ξ_0, ξ_1, σ and α are fixed positive. Equation (1.1) is related to the flutter panel equation with memory term. This equation arises in a wind tunnel experiment for a panel at supersonic speeds. For a derivation of this model see, for instance, Dowell [14], Holmes [15, 16], Bass [8] and Balakrishnan and Taylor [7].

It is well known that viscoelastic materials exhibit natural damping, which is due to the special property of these materials to keep memory of their past history. From the mathematical point of view, these damping effects are modeled by integrodifferential operators. A simple example is the viscoelastic membrane equation

$$u_{tt} - \Delta u + \int_{-\infty}^{t} h(t-s)\,\Delta u \mathrm{d}s = 0.$$

Therefore, the dynamics of viscoelastic materials are interesting and of great importance, as these materials have a wide application in natural sciences. Equations of type (1.1) are a class of essential nonlinear wave equations describing the spread of strain waves in a viscoelastic configuration. In the case of space dimension $n = 1$ and $M(t) = 1$, the equations in (1.1) describe the spread of strain waves in a viscoelastic bar made up of the material of the rate type. They can also be seen as field equations governing the longitudinal motion of a viscoelastic bar.

In the absence of the viscoelastic term ($h = 0$) and $\sigma = 0$, we have the well known Kirchhoff equation

$$u_{tt} - \left(\xi_0 + \xi_1 \|\nabla u(t)\|_2^2\right) \Delta u = 0,$$

it was extensively studied by several authors in both $(1, 2, \cdots, n)$-dimensional cases and general mathematical models in a Hilbert space H. Both local and global solutions were shown to exist in several physical-mathematical contexts. Among them, Arosio-Spagnolo [6], Ono [22–24], Clark [13], Kirchhoff [18], Narashinham [21], Pohozaev [25] and a number of other interesting references cited in the previously mentioned papers, namely, in Cavalcanti et al. [1, 2, 9–12]. There exists a large body of literature on viscoelastic problems with the memory term. Among these we can cite Appleby et al. [5], Rammaha et al. [26], and Vicente et al. [27]. Lu et al. [20] considered the following problem:

$$\begin{cases} u_{tt} - \Delta u + \displaystyle\int_0^t h(t-s)\,\Delta u \mathrm{d}s = 0 & \text{in } \Omega \times (0, +\infty), \\ u(x,0) = u_0(x) \ \text{ and } u_t(x,0) = u_1(x) & \text{in } \bar{\Omega}, \\ u = 0 & \text{on } \Gamma_1 \times [0, +\infty), \\ \dfrac{\partial u}{\partial \nu} - \displaystyle\int_0^t h(t-s)\,\frac{\partial u}{\partial \nu}\mathrm{d}s + |u_t|^{m-2} u_t = |u|^{p-2} u & \text{on } \Gamma_0 \times [0, +\infty). \end{cases}$$

The authors proved local existence of the solutions in the energy space when $m > \frac{r}{r+1-p}$ or $n = 1, 2$, where $r = \frac{2(n-1)}{n-2}$, and global existence when $p \leq m$ or the initial data was