Available online at www.sciencedirect.com

ScienceDirect

Acta Mathematica Scientia 2013,**33B**(2):578–588

Mathematica Acta Scientia 数学物理学报

http://actams.wipm.ac.cn

ATTRACTING AND QUASI-INVARIANT SETS OF STOCHASTIC NEUTRAL PARTIAL FUNCTIONAL DIFFERENTIAL EQUATIONS*

Dingshi LI (黎定仕)
School of Mathematics, Southwest Jiaotong University, Chengdu 610031, China
E-mail: *lidingshi2006@163.com*

Daoyi XU (徐道义)
Yangtze Center of Mathematics, Sichuan University, Chengdu 610064, China
E-mail: *dyxu@scu.edu.cn*

Abstract In this article, we investigate a class of stochastic neutral partial functional differential equations. By establishing new integral inequalities, the attracting and quasi-invariant sets of stochastic neutral partial functional differential equations are obtained. The results in [15, 16] are generalized and improved.

Key words Attracting; quasi-invariant; neutral; stochastic

2010 MR Subject Classification 35B35; 35B40; 39B82

1 Introduction

The attracting set and the invariant set of dynamical systems have been extensively studied over the past few decades and various results are reported. For discrete systems, see [1–3]. For deterministic differential systems with or without delays, see [4–8]. For partial differential systems, see [9]. For stochastic or random systems, see [10, 11].

Stochastic partial differential equations in Hilbert spaces were studied by some authors and many valuable results on the existence, uniqueness and stability of the solutions were established; see, for example, [12–22]. However, to the best of our knowledge, there exist only a few articles which dealt with the attracting set and the invariant set of stochastic partial functional differential equations. To be more precise, treated within a variation formulation in [14], it is analyzed the almost sure exponential stability and ultimate boundedness of the solutions to a class of neutral stochastic semilinear partial delay differential equations.

In this article, on the basis of the above articles, we consider some stochastic neutral partial functional differential equations. By establishing new integral inequalities, the attracting and quasi-invariant sets of stochastic neutral partial functional differential equations are obtained. The related known results in [15, 16] are generalized and improved.

*Received October 31, 2011; revised April 27, 2012. The first author is supported by National Natural Science Foundation of China (11271270 and 11201320).

2 Model Description and Preliminaries

Throughout this article, H and K will denote two real separable Hilbert spaces and we denote by $\langle\cdot,\cdot\rangle_H$, $\langle\cdot,\cdot\rangle_K$ their inner products and by $\|\cdot\|_H$, $\|\cdot\|_K$ their vector norms, respectively. Let $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq 0},P)$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions (that is, it is right continuous and $\mathscr{F}_0$ contains all P-null sets). We denote by $\mathcal{L}(K,H)$ the set of all linear bounded operators from K into H, equipped with the usual operator norm $\|\cdot\|$. In this article, we always use the same symbol $\|\cdot\|$ to denote norms of operators regardless of the spaces potentially involved when no confusion possibly arises. $E[f]$ means the mathematical expectation of f

Let $\{W(t),t\geq 0\}$ denote a K-valued $\{\mathscr{F}_t\}_{t\geq 0}$-Wiener process defined on $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq 0},P)$ with covariance operator Q, that is,

$$E\langle W(t),x\rangle_K\langle W(s),y\rangle_K=(t\wedge s)\,\langle Qx,y\rangle_K\quad\text{for all}\quad x,y\in K,$$

where Q is a positive, self-adjoint, trace class operator on K. In particular, we shall call such $W(t)$, $t\geq 0$, a K-valued Q-Wiener process with respect to $\{\mathscr{F}_t\}_{t\geq 0}$.

To define stochastic integrals with respect to the Q-Wiener process $W(t)$, we introduce the subspace $K_0=Q^{1/2}(K)$ of K which, endowed with the inner product $\langle u,v\rangle_{K_0}=\langle Q^{-1/2}u,$ $Q^{-1/2}v\rangle_K$, is a Hilbert space. Let $\mathcal{L}_2^0=\mathcal{L}_2(K_0,H)$ denote the space of all Hilbert-Schmidt operators from K_0 into H. It turns out to be a separable Hilbert space, equipped with the norm

$$\|\psi\|^2_{\mathcal{L}_2^0}=\operatorname{tr}\left(\left(\psi Q^{1/2}\right)\left(\psi Q^{1/2}\right)^*\right)\quad\text{for all}\quad\psi\in\mathcal{L}_2^0.$$

Clearly, for any bounded operators $\psi\in\mathcal{L}(K,H)$, this norm reduces to $\|\psi\|^2_{\mathcal{L}_2^0}=\operatorname{tr}(\psi Q\psi^*)$. The reader is referred to Da Prato and Zabczyk [23] for a systematic theory about stochastic integrals of this kind.

$R_+=[0,+\infty)$ and $C(X,Y)$ denotes the space of continuous mappings from the topological space X to the topological space Y. Especially, let $C\triangleq C([-\tau,0],R)$, where τ is a positive constant. For $\phi\in C$, we denote $|\phi(t)|_\tau=\sup\limits_{-\tau\leq s\leq 0}|\phi(t+s)|$. Denote $C_H=C([-\tau,0],H)$ equipped with the norm $\|\phi\|_{C_H}=\sup\limits_{-\tau\leq s\leq 0}\|\phi\|_H$. Denote by $BC^b_{\mathscr{F}_0}([-\tau,0],H)$ the family of all bounded $\mathscr{F}_0$-measurable, C_H-valued random variables ϕ, satisfying $\|\phi\|^p_{L^p}=\sup\limits_{-\tau\leq s\leq 0}E\|\phi(s)\|^p_H<\infty$.

Consider stochastic neutral partial functional differential equations:

$$\begin{cases} d\left[x(t)+f(t,x_t)\right]=\left(Ax(t)+g(t,x_t)\right)\mathrm{d}t+\sigma(t,x_t)\,\mathrm{d}W(t), & t\geq 0,\\ x_0(s)=\varphi\in BC^b_{\mathscr{F}_0}([-\tau,0],H), & s\in[-\tau,0], \end{cases}\tag{2.1}$$

where $A:D(A)\subset H\to H$ is the infinitesimal generator of an analytic semigroup of linear operator $(T(t))_{t\geq 0}$ on a Hilbert space H. $f,\ g:\ [0,\infty)\times C_H\to H$ and $\sigma:[0,\infty)\times C_H\to\mathcal{L}_2^0$ are jointly continuous functions.

We also assume $0\in\rho(-A)$. Then, it is possible to define the fractional power $(-A)^\alpha$ for some $0<\alpha\leq 1$ as a closed linear operator with its domain $D((-A)^\alpha)$. Furthermore, we have the following properties appeared in [24].

Remark 2.1 Suppose $0\in\rho(-A)$, then by Theorem 6.13, p. 74, in [24] we prove that there exist positive constants M,γ such that $\|T(t)\|\leq Me^{-\gamma t}$ for $t\geq 0$.