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SEPARATION PROPERTY OF POSITIVE RADIAL SOLUTIONS FOR A GENERAL SEMILINEAR ELLIPTIC EQUATION*

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Abstract The asymptotic behavior at infinity and an estimate of positive radial solutions of the equation

$$\Delta u+\sum_{i=1}^{k}c_i r^{l_i}u^{p_i}=0, \qquad x\in\mathbb{R}^n, \tag{0.1}$$

are obtained and the structure of separation property of positive radial solutions of Eq. (0.1) with different initial data α is discussed.

Key words Asymptotic behavior; separation property; semilinear elliptic equation

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1 Introduction

In this article, we study the general semilinear elliptic equation

$$\Delta u+\sum_{i=1}^{k}c_i|x|^{l_i}u^{p_i}=0, \qquad x\in\mathbb{R}^n, \tag{1.1}$$

where $n\geq 3,\ c_i>0,\ l_i>-2,\ p_i>1,\ i=1,2,\cdots,k$, and $\Delta=\sum\limits_{i=1}^{n}\frac{\partial^2}{\partial x_i^2}$ is the n-dimension Laplacian.

Eq. (1.1) has its origin from, e.g., the prescribed curvature problems in Riemannian geometry and astrophysics (that is, the Lane-Emden-Fowler equation and the Matukuma equation as special cases). For some reasons, many mathematicians considered the radial solutions. Under suitable conditions, radial solutions to (1.1) must oscillate about the zero, see e.g., [7, 12, 19, 20]. In [7, 12], the authors studied the asymptotic behavior of the period between two consecutive zeros of solutions of Eq. (1.1) when $k=2$ under certain assumptions on p_i and $l_i,\ i=1,\ 2.$

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However, the separation property and asymptotic behavior of the positive solutions to (1.1) (homogeneous and inhomogeneous) when $k=1$ received much more attention, see e.g., [1, 3–6, 8, 9, 11, 14–19] and the reference therein.

For $l_i > -2$, at $x=0$, $c_i r^{l_i},\ r=|x|,\ i=1,\ 2,\ 3,\cdots,k$, are bad usually, we cannot expect the solutions to be differentiable, or even continuous owing to the singularity of $c_i r^{l_i},\ i=1,\ 2,\ 3,\cdots,k$, at $x=0$. Let u be a solution of Eq. (1.1), the singular point $x=0$ is called a removable singular point of $u(x)$ if $u(0)\equiv \lim\limits_{x\to 0} u(x)$ exists, otherwise, $x=0$ is called a nonremovable singular point. It is showed by Ni and Yotsutani that, when $x=0$ is a removable singular point of a regular solution, the existence of the derivatives of the solution depends on the "blow-up" rate of $c_i r^{l_i},\ i=1,\ 2,\ 3,\cdots,k$, at $x=0$.

Definition 1 Let $u\in C^2(\mathbb{R}^n/\{0\})$ be a solution of Eq. (1.1). If $x=0$ is a removable singular point of u, then, u is said to be a regular solution of Eq. (1.1), otherwise, u is said to be an irregular solution of Eq. (1.1).

For the following Eq. (1.2), we have the similar definition. Without any particular statement, all solutions considered in this article are regular ones.

When $k=1$, many authors considered the radial form of Eq. (1.1):

$$u''+\frac{n-1}{r}u'+K(r)u^p=0,\qquad r\geq 0, \tag{1.2}$$

where K is a continuous function in $\mathbb{R}$, $\sigma>-2$ and $\lim\limits_{r\to\infty} r^{-\sigma}K(r)=k_\infty$. For convenience, we denote $m_\sigma=\frac{2+\sigma}{p-1}$ and $L_\sigma=(m_\sigma(n-2-m_\sigma))^{1/(p-1)}$. The asymptotic behavior of positive radial solutions was extensively studied. Especially, Li obtained in [14] that, when $p>\frac{n+\sigma}{n-2}$,

$$\lim_{r\to\infty} r^{m_\sigma}u(r)=u_\infty=\begin{cases} L_\sigma/k_\infty^{1/(p-1)} & \text{or}\\ 0. & \end{cases}$$

under certain differentiable and integral assumptions on K. Wang obtained that, when $K(r)=r^\sigma$ and $p>\frac{n+2+2\sigma}{n-2}$, $\lim\limits_{r\to\infty} r^{m_\sigma}u(r)=L_\sigma$ in [21]. Then, they considered the initial problem

$$\begin{cases} u''+\frac{n-1}{r}u'+K(r)u^p=0, \qquad r>0,\\ u(0)=\alpha>0. \end{cases}$$

With additional non-increasing assumption on $r^{-\sigma}K(r)$, Liu, Li and Deng, and Bae and Chang obtained that $r^{m_\sigma}u(r)$ is increasing reseptively. On the basis of the asymptotic behavior of positive solutions, they studies further the structure of separation and intersection of positive solutions respectively and obtained that, when $p>p_c(\sigma)$, any two positive solutions does not intersect each other (see Theorem 1(i) in [16] and Theorem 1.2 in [4]); while when $\frac{n+2+2\sigma}{n-2}<p<p_c(\sigma)$, any two positive solutions will intersect infinity many times (see Theorem 1(ii) in [16] and Theorem 1.1 in [2]).

For the inhomogeneous case, for example,

$$\begin{cases} u''+\frac{n-1}{r}u'+K(r)u^p+\mu f(r)=0, \qquad r>0,\\ u(0)=\alpha>0, \end{cases}$$