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Fast computation of Hessian-based enhancement filters for medical images

Shih-Feng Yang[a,*], *Ching-Hsue Cheng*[b]

[a] *Department of Information Management, Transworld University, 1221 Zhennan Rd., Douliu City, Yunlin County 640, Taiwan*

[b] *Department of Information Management, National Yunlin University of Science & Technology, 123, Sec. 3, University Rd., Douliu City, Yunlin County 640, Taiwan*

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ABSTRACT

This paper presents a method for fast computation of Hessian-based enhancement filters, whose conditions for identifying particular structures in medical images are associated only with the signs of Hessian eigenvalues. The computational costs of Hessian-based enhancement filters come mainly from the computation of Hessian eigenvalues corresponding to image elements to obtain filter responses, because computing eigenvalues of a matrix requires substantial computational effort. High computational cost has become a challenge in the application of Hessian-based enhancement filters. Using a property of the characteristic polynomial coefficients of a matrix and the well-known Routh–Hurwitz criterion in control engineering, it is shown that under certain conditions, the response of a Hessian-based enhancement filter to an image element can be obtained without having to compute Hessian eigenvalues. The computational cost can thus be reduced. Experimental results on several medical images show that the method proposed in this paper can reduce significantly the number of computations of Hessian eigenvalues and the processing times of images. The percentage reductions of the number of computations of Hessian eigenvalues for enhancing blob- and tubular-like structures in two-dimensional images are approximately 90% and 65%, respectively. For enhancing blob-, tubular-, and plane-like structures in three-dimensional images, the reductions are approximately 97%, 75%, and 12%, respectively. For the processing times, the percentage reductions for enhancing blob- and tubular-like structures in two-dimensional images are approximately 31% and 7.5%, respectively. The reductions for enhancing blob-, tubular-, and plane-like structures in three-dimensional images are approximately 68%, 55%, and 3%, respectively.

1. Introduction

Image pre-processing plays an important role in medical image processing. Many pre-processing techniques have been developed [1–3]. Among those techniques, the Hessian-based enhancement filter is one of the most widely used. Hessian-based enhancement filters have been developed and adopted to process blob-, tubular-, and plane-like structures in medical images [4–15].

* Corresponding author. Tel.: +886 5 5370988 ext 8110; fax: +886 5 5370989.
E-mail addresses: ysf@twu.edu.tw, ysf16345@gmail.com (S.-F. Yang).
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High computational cost is one of the main drawbacks of Hessian-based enhancement filters [12,16,17]. The Hessian-based enhancement filter technique is based on the eigenvalue analysis of the Hessian matrix corresponding to each image element in two-dimensional (2D) or three-dimensional (3D) images. To obtain the response of a Hessian-based enhancement filter to an image element, the Hessian eigenvalues corresponding to the element are first computed and arranged in some order. The filter response is then obtained by computing the designed response function with the arranged Hessian eigenvalues. Computing eigenvalues of a matrix requires substantial computational effort. Moreover, Hessian-based enhancement filters are commonly applied in multiscale frameworks. One has to compute Hessian eigenvalues for each image element for every selected scale. Consequently, high computational cost becomes a challenge in the application of Hessian-based enhancement filters.

Few efforts have been dedicated to solving the problem of high computational cost in the application of Hessian-based enhancement filters. Orlowski and Orkisz [17] proposed a method to reduce the computational costs of Hessian-based enhancement filters for tubular-like structures in 3D images. Their method is based on the fact that the response of a Hessian-based enhancement filter to an image element is zero if the specified condition in the response function of the filter for identifying a particular structure is not satisfied. They showed that for a voxel, one can conclude by performing two tests that the condition for identifying tubular-like structures is not satisfied. The first test checks the sign of the Hessian trace, and the second test examines the sign of the Hessian determinant and whether the Hessian matrix is negative-definite. If the two tests conclude that the condition is not satisfied, the response of the Hessian-based enhancement filter to the voxel can be determined to be zero without having to compute the corresponding Hessian eigenvalues. The computational cost of the Hessian-based enhancement filter can thus be reduced.

In this paper, a method is proposed for reducing the computational costs of Hessian-based enhancement filters for blob-, tubular-, and plane-like structures in 2D and 3D images. It is shown that for an image element, under certain conditions, we can conclude that the conditions in the response functions of the Hessian-based enhancement filters are not satisfied without having to compute the corresponding Hessian eigenvalues. These conditions are developed by using a property of the characteristic polynomial coefficients of the Hessian matrix corresponding to the image element and the well-known Routh–Hurwitz criterion in control engineering. They can be checked with very low computational costs. As a result, the number of computations of Hessian eigenvalues required by the filters to process images can be reduced with very low computational cost. The computational costs of the filters are thereby reduced.

The rest of the paper is organized as follows. Section 2 reviews Hessian-based enhancement filters for 2D and 3D images. Section 3 develops the method for reducing the computational costs of Hessian-based enhancement filters for blob-, tubular-, and plane-like structures in 2D and 3D images. The experimental results are given in Section 4 to show the effectiveness of the method. The results are discussed in Section 5, and conclusions are drawn in Section 6.

2. Hessian-based enhancement filters

Let $f(\mathbf{x})$ be the intensity function of an image, where $\mathbf{x}$ is the location in the image space. For a 2D image, the Hessian matrix, $\mathbf{H}_2(\mathbf{x})$, corresponding to the pixel at $\mathbf{x} = (x,y)$ is given by

$$\mathbf{H}_2(\mathbf{x}) = \begin{bmatrix} \dfrac{\partial^2 f}{\partial x^2} & \dfrac{\partial^2 f}{\partial x \partial y} \\ \dfrac{\partial^2 f}{\partial y \partial x} & \dfrac{\partial^2 f}{\partial y^2} \end{bmatrix} \triangleq \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} \tag{1}$$

where $\frac{\partial^2 f}{\partial x^2}$, f_{xx}, $\frac{\partial^2 f}{\partial x \partial y}$, f_{xy}, etc. are the second order partial derivatives of f.

Similarly, for a 3D image, the Hessian matrix, $\mathbf{H}_3(\mathbf{x})$, corresponding to the voxel at $\mathbf{x} = (x,y,z)$ is given by

$$\mathbf{H}_3(\mathbf{x}) = \begin{bmatrix} \dfrac{\partial^2 f}{\partial x^2} & \dfrac{\partial^2 f}{\partial x \partial y} & \dfrac{\partial^2 f}{\partial x \partial z} \\ \dfrac{\partial^2 f}{\partial y \partial x} & \dfrac{\partial^2 f}{\partial y^2} & \dfrac{\partial^2 f}{\partial y \partial z} \\ \dfrac{\partial^2 f}{\partial z \partial x} & \dfrac{\partial^2 f}{\partial z \partial y} & \dfrac{\partial^2 f}{\partial z^2} \end{bmatrix} \triangleq \begin{bmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{bmatrix} \tag{2}$$

For convenience and without causing ambiguity, let $\Lambda = (\lambda_1, \lambda_2)$ with $|\lambda_1| \geq |\lambda_2|$ and $\Lambda = (\lambda_1, \lambda_2, \lambda_3)$ with $|\lambda_1| \geq |\lambda_2| \geq |\lambda_3|$ be the eigenvalues of $\mathbf{H}_2(\mathbf{x})$ and $\mathbf{H}_3(\mathbf{x})$, respectively. The response function, $\Phi(\mathbf{x})$, of a Hessian-based enhancement filter has the form

$$\Phi(\mathbf{x}) = \begin{cases} \phi(\Lambda), & \text{if condition } \Omega_\Lambda \text{ is satisfied;} \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

In designing the response function $\Phi(\mathbf{x})$, the condition Ω_Λ is specified according to the structure to be enhanced. The image element at $\mathbf{x}$ is determined to belong to the structure if the Hessian eigenvalues corresponding to the image element satisfy the condition Ω_Λ. If this is the case, the filter response to the image element is given by the designed function, $\phi(\Lambda)$; otherwise, the filter response is zero.

The condition Ω_Λ in the response function $\Phi(\mathbf{x})$ in Eq. (3) is associated only with the signs of Hessian eigenvalues. The derivation of the condition Ω_Λ for identifying blob-, tubular-, and plane-like bright structures in a dark background can be found in [9,12]. Typically, the conditions, $\Omega_{\Lambda,b2}$ and $\Omega_{\Lambda,t2}$, for identifying blob- and tubular-like bright structures, respectively, in 2D images and the conditions, $\Omega_{\Lambda,b3}$, $\Omega_{\Lambda,t3}$, and $\Omega_{\Lambda,p3}$,