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WEIGHTED NORM INEQUALITIES FOR THE COMMUTATORS OF MULTILINEAR SINGULAR INTEGRAL OPERATORS*

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Abstract In this paper, the authors consider the weighted estimates for the commutators of multilinear Calderón-Zygmund operators. By introducing an operator which shifts the commutation, and establishing the weighted estimates for this new operator, the authors prove that, if $p_1 \in (1,\infty)$, $p_2,\cdots,p_m \in (1,\infty]$, $p \in (0,\infty)$ with $1/p = \sum\limits_{1\le k\le m} 1/p_k$, then for any weight w, the commutators of m-linear Calderón-Zygmund operator are bounded from $L^{p_1}(\mathbb{R}^n, M_{L(\log L)^\sigma}w) \times L^{p_2}(\mathbb{R}^n, Mw) \times \cdots \times L^{p_m}(\mathbb{R}^n, Mw)$ to $L^p(\mathbb{R}^n, w)$ with σ to be a constant depending only on p_1 and the order of commutator.

Key words multilinear Calderón-Zygmund operator; weighted norm inequality; commutator; Calderón-Zygmund decomposition

2000 MR Subject Classification 42B20

1 Introduction

In recent years, considerable attention was paid to the study of the multilinear Calderón-Zygmund operators. The original works in this area were the papers of Coifman and Meyer [2, 3]. Let $m \ge 1$, $K(x; y_1, \cdots, y_m)$ be a locally integrable function defined away from the diagonal $x = y_1 = y_2 = \cdots = y_m$ in $(\mathbb{R}^n)^{m+1}$, $A > 0$ and $\gamma \in (0, 1]$ be two constants. We say that K is a kernel in m-CZK(A, γ) if it satisfies the size condition that for all $(x, y_1, \cdots, y_m) \in (\mathbb{R}^n)^{m+1}$ with $x \ne y_j$ for some $1 \le j \le m$,

$$|K(x; y_1, \cdots, y_m)| \le \frac{A}{(|x-y_1| + \cdots + |x-y_m|)^{mn}} \tag{1.1}$$

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and satisfies the regularity condition

$$|K(x;\, y_1,\cdots,y_m)-K(x';\, y_1,\cdots,y_m)|\le \frac{A|x-x'|^\gamma}{(|x-y_1|+\cdots+|x-y_m|)^{mn+\gamma}}, \tag{1.2}$$

whenever $\max\limits_{1\le k\le m}|x-y_k|\ge 2|x-x'|$, and also that, for each fixed k with $1\le k\le m$,

$$|K(x;\, y_1,\cdots,y_k,\cdots,y_m)-K(x;\, y_1,\cdots,y_k',\cdots,y_m)|\le \frac{A|y_k-y_k'|^\gamma}{(|x-y_1|+\cdots+|x-y_m|)^{mn+\gamma}}, \tag{1.3}$$

whenever $\max\limits_{1\le j\le m}|x-y_j|\ge 2|y_k-y_k'|$. An operator T, defined on m-fold product of Schwartz spaces and taking values in the space of tempered distributions, is said to be an m-linear Calderón-Zygmund operator with kernel K if

(a) T is m-linear,

(b) For some $q_1,\cdots,q_m\in[1,\infty]$ and $q\in(0,\infty)$ with $1/q=\sum\limits_{k=1}^m 1/q_k$, T can be extended to be a bounded operator from $L^{q_1}(\mathbb{R}^n)\times L^{q_2}(\mathbb{R}^n)\times\cdots\times L^{q_m}(\mathbb{R}^n)$ to $L^q(\mathbb{R}^n)$,

(c) For $f_1,\cdots,f_m\in L^2(\mathbb{R}^n)$ with compact supports, and for $x\notin\bigcap\limits_{k=1}^m \operatorname{supp} f_k$,

$$T(f_1,\cdots,f_m)(x)=\int_{(\mathbb{R}^n)^m}K(x;\, y_1,\cdots,y_m)\prod_{k=1}^m f_k(y_k)\mathrm{d}y_1\cdots\mathrm{d}y_m \tag{1.4}$$

and K is in m-CZK(A,γ) for some constants A and γ.

It is obvious that when $m=1$, this operator is just the classical Calderón-Zygmund operator. In the case of $m\ge 2$, this operator has intimate connections with operator theory and partial differential equations, and was considered first by Coifman and Meyer, [2], [3], and then by many authors. In their remarkable work [8], Grafakos and Torres considered the mapping properties of T on the space of type $L^{p_1}(\mathbb{R}^n)\times\cdots\times L^{p_m}(\mathbb{R}^n)$ with $1\le p_1,\cdots,p_m\le\infty$, and established a $T1$ type theorem for the operator T. Also, Grafakos and Torres [9] established the weighted estimates with A_p weights for T and the maximal operator associated with T^*. For other works about the multilinear Calderón-Zygmund operator, see [6], [9], [10], [17], and [18].

Now, let b be a BMO$(\mathbb{R}^n)$ function. For a fixed positive integer k with $1\le k\le m$, define the operator $[b,T]^k$ by

$$[b,T]^k(f_1,\cdots,f_m)(x)=b(x)T(f_1,\cdots,f_k,\cdots,f_m)(x)-T(f_1,\cdots,bf_k,\cdots,f_m)(x).$$

Let $b_1,\cdots,b_m$ be BMO$(\mathbb{R}^n)$ functions and $\vec{b}=(b_1,\cdots,b_m)$. Define the commutator $T^{\vec{b}}$ by

$$T^{\vec{b}}(f_1,\cdots,f_m)(x)=\sum_{1\le k\le m}[b_k,T]^k(f_1,\cdots,f_m)(x). \tag{1.5}$$

The operator $T^{\vec{b}}$ was introduced by Pérez and Torres [17]. By $A_p(\mathbb{R}^n)$ estimate of the operator T, Pérez and Torres [17] proved that $T^{\vec{b}}$ is bounded from $L^{p_1}(\mathbb{R}^n)\times\cdots\times L^{p_m}(\mathbb{R}^n)$ to $L^p(\mathbb{R}^n)$ for any $p_1,\cdots,p_m\in(1,\infty)$ and $p\in(1,\infty)$ with $1/p=\sum\limits_{k=1}^m 1/p_k$. Recently, Lerner et al. [13] introduced a new maximal operator, and then extended the result in [17] to the case