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HÖLDER CONTINUOUS SOLUTIONS FOR SECOND ORDER INTEGRO-DIFFERENTIAL EQUATIONS IN BANACH SPACES*

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Abstract We study Hölder continuous solutions for the second order integro-differential equations with infinite delay (P_1): $u''(t)+cu'(t)+\int_{-\infty}^t \beta(t-s)u'(s)\mathrm{d}s+\int_{-\infty}^t \gamma(t-s)u(s)\mathrm{d}s = Au(t) - \int_{-\infty}^t \delta(t-s)Au(s)\mathrm{d}s + f(t)$ on the line $\mathbb{R}$, where $0 < \alpha < 1$, A is a closed operator in a complex Banach space X, $c \in \mathbb{C}$ is a constant, $f \in C^\alpha(\mathbb{R}, X)$ and $\beta, \gamma, \delta \in L^1(\mathbb{R}_+)$. Under suitable assumptions on the kernels β, γ and δ, we completely characterize the C^α-well-posedness of (P_1) by using operator-valued $\dot{C}^\alpha$-Fourier multipliers.

Key words Fourier multiplier; C^α-well-posedness; integro-differential equations

2000 MR Subject Classification 45N05; 45D05; 43A15; 47D99

1 Introduction

In a series of recent publications operator-valued Fourier multipliers on vector-valued function spaces were studied (see e.g. [1–3, 5, 14, 15]). They are needed to study the existence and uniqueness of solutions for differential equations on Banach spaces. In this paper, we use operator-valued $\dot{C}^\alpha$-multiplier results established in [3] to study the C^α-well-posedness for the following integro-differential equations with infinite delay:

$$(P_1) \qquad u''(t) + cu'(t) + \int_{-\infty}^t \beta(t-s)u'(s)\mathrm{d}s + \int_{-\infty}^t \gamma(t-s)u(s)\mathrm{d}s = Au(t) - \int_{-\infty}^t \delta(t-s)Au(s)\mathrm{d}s + f(t) \quad (t \in \mathbb{R}),$$

here $0 < \alpha < 1$, A is a closed operator in a complex Banach space X, $c \in \mathbb{C}$ is a constant, $f \in C^\alpha(\mathbb{R}, X)$ and $\beta, \gamma, \delta \in L^1(\mathbb{R}_+)$. In this paper, under suitable assumptions on the kernels β, γ and δ, we are able to completely characterize the C^α-well-posedness of (P_1).

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We recall that, when $\gamma = 0$, the same second order integro-differential equations with infinite delay on the interval $[0, 2\pi]$ with periodic boundary conditions

$$(\mathrm{P}_2) \quad \begin{cases} \quad u''(t) + cu'(t) + \displaystyle\int_{-\infty}^{t} \beta(t-s)u'(s)\mathrm{d}s \\ = Au(t) - \displaystyle\int_{-\infty}^{t} \delta(t-s)Au(s)\mathrm{d}s + f(t) \quad (0 \le t \le 2\pi), \\ \quad u(0) = u(2\pi), u'(0) = u'(2\pi) \end{cases}$$

were studied by Bu and Fang [6], where they gave conditions on the kernels β and δ to ensure the L^p-well-posedness, $B^s_{p,q}$-well-posedness or $F^s_{p,q}$-well-posedness of (P_2).

Many literatures devoted to the similar first order integro-differential equation

$$(\mathrm{P}_3) \quad \begin{cases} \quad \gamma_0 u'(t) + \displaystyle\int_{-\infty}^{t} b(t-s)u'(s)\mathrm{d}s + \gamma_\infty u(t) \\ = c_0 Au(t) - \displaystyle\int_{-\infty}^{t} a(t-s)Au(s)\mathrm{d}s + f(t) \quad (0 \le t \le 2\pi), \\ \quad u(0) = u(2\pi), \end{cases}$$

where $\gamma_0, \gamma_\infty, c_0$ are constants, A is a closed linear operator in X, and $a, b \in L^1(\mathbb{R}_+)$. The class of equation of type (P_1), (P_2) and (P_3) arises as models for nonlinear heat conduction in material of fading memory type and population dynamics. In [12], Keyantuo and Lizama obtained the maximal regularity of (P_3) on L_p spaces and Besov spaces. They also studied this equation in the case $\gamma_0 = c_0 = 1, b = \gamma_\infty = 0$ in a previous paper [11]. Clément and Da Prato studied (P_3) on the real line in the case $a = 0$ and obtained maximal regularity results in Sobolev spaces and Hölder spaces as well as the space of bounded uniformly continuous functions [8]. Da Prato and Lunardi [9] investigated periodic solutions of equation (P_3) in the case $b = 0$. Hölder continuous solutions for equation (P_3) were studied on the real line by Lunardi [13] in the case A to be the Laplacian operator in a bounded domain $\Omega \subset \mathbb{R}^N$ and $X = C(\bar{\Omega})$. Recently, under some mild conditions on a, b, Keyantuo and Lizama were able to completely characterize the C^α-well-posedness of (P_2) [10].

We notice that problem (P_1) was studied by several authors in a more simpler form and different boundary conditions. For instance, Chill and Srivastava [7] considered the L^p-maximal regularity on a finite interval [0,T) for the abstract second order problem

$$(\mathrm{P}_4) \quad \begin{cases} u''(t) + Bu'(t) + Au(t) = f(t) \quad (0 \le t < T), \\ u(0) = 0, \ u'(0) = 0. \end{cases}$$

The semigroup theory and the trace spaces played important roles in their discussions. Under a suitable condition on the operators A and B, they gave a necessary and sufficient condition for problem (P_4) to have the L^p-maximal regularity.

In this paper, we are interested in the existence and uniqueness of Hölder continuous solutions of (P_1). Since A is not necessarily the generator of semigroups in our situation, semigroup theory is no longer applicable. So our main tool in this paper is the operator-valued $\dot{C}^\alpha$-Fourier multiplier results established by Arendt, Batty and Bu [3]. The conditions that we