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ON THE C. CHANG TYPE INEQUALITY OF ALGEBROID FUNCTIONS*

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Abstract In this paper, we investigate the growth relations between algebroid functions and their derivatives, and extend famous C. Chang inequality (see [1, 4]) of meromorphic functions to algebroid functions.

Key words algebroid functions; derivative; characteristic function

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1 Introduction and Main Result

Let $A_k(z), \cdots, A_0(z)$ be entire functions without common zeros, then the equation

$$\Psi(z, W) = A_k(z)W^k + A_{k-1}(z)W^{k-1} + \cdots + A_0(z) = 0 \tag{1}$$

defines a k-valued algebroid function $W(z)$ in $|z| < \infty$ (see [3, 5]). If $A_k(z) = 1$, then $W(z)$ is called a k-valued entire algebroid function. If $A_k(z), \cdots, A_0(z)$ all are polynomials, then $W(z)$ is called a k-valued algebraic function (we generally consider $\{A_j(z)\}$ containing at least one transcendental entire function). Let M denote the field of meromorphic functions, we use the notation $M[x]$ for the ring of polynomials with coefficients in M. If $\Psi(z, W) \in M[x]$ is irreducible over M, then $W(z)$ is called a k-valued irreducible algebroid function.

Let $W(z)$ be a k-valued irreducible algebroid function defined by (1) in $|z| < \infty$. If $A_k(z_0) \neq 0$, and if $\Psi(z_0, W) = 0$ and its partial derivative $\Psi_w(z_0, W) = 0$ have no common roots, then z_0 is said to be a regular point of $W(z)$. The set of all regular points of $W(z)$ is called the regular set, denoted by T_W. Its complementary set $S_W := \mathbf{C} - T_W$ is called the critical set of $W(z)$. We know that each critical point $z_0 \in S_W$ is an isolated point. The single valued domain of the irreducible k-valued algebroid function $W = W(z)$ is the connected Riemann surface $\widetilde{T}_z$, the point on which is a regular function element $\widetilde{b} = (w_{b,j}(z), b) := \{(w_{b,j}(z), B(b, r)\}$. For

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any two regular function elements $(w_{a,j}(z), a)$ and $(w_{b,j}(z), b)$ on $\widetilde{T}_z$, there always exists a path $\gamma \subset \widetilde{T}_z$ along which they can be continuated analytically to each other (see [5]). We generally write $W = W(z) = \{(w_j(z), b)\}_{j=1}^k$.

Definition 1 Let $W(z) = \{(w_j(z), a)\}_{j=1}^k$ be a k-valued algebroid function defined by (1), $W'(z) := \{(w'_j(z), a)\}$ is called the derivative of $W(z)$, if

$$\begin{aligned}\Psi'(z, W') &= B_k(z)(W' - w'_1(z))(W' - w'_2(z)) \cdots (W' - w'_k(z)) \\ &= B_k(z)(W')^k + B_{k-1}(z)(W')^{k-1} + \cdots + B_0(z) = 0, \end{aligned} \tag{2}$$

where $B_k(z), \cdots, B_0(z)$ are entire functions without common zeros.

We now recall the basic notations of Nevanlinna value distribution theory of algebroid functions (see [3]).

For the proximity function, we define, for $a \in \mathbf{C}$,

$$m(r, \frac{1}{W-a}) = m(r, a) := \frac{1}{2k\pi} \sum_{j=1}^{k} \int_0^{2\pi} \log^+ \frac{1}{|w_j(re^{i\theta}) - a|} d\theta$$

and for $a = \infty$,

$$m(r, W) = m(r, \infty) := \frac{1}{2k\pi} \sum_{j=1}^{k} \int_0^{2\pi} \log^+ |w_j(re^{i\theta})| d\theta = \frac{1}{k} \sum_{j=1}^{k} m(r, w_j).$$

Let $n(r, a)$ be the number of a-points of $W(z)$, counted according to their multiplicity, in the disk $|z| < r$, we write

$$N\Big(r, \frac{1}{W-a}\Big) = N(r, a) := \frac{1}{k} \int_0^r \frac{n(t, a) - n(0, a)}{t} dt + \frac{1}{k} n(0, a) \log r$$

for $a \in C$, correspondingly,

$$N(r, W) = N(r, \infty) := \frac{1}{k} \int_0^r \frac{n(t, W) - n(0, W)}{t} dt + \frac{1}{k} n(0, W) \log r$$

for $a = \infty$. Defining the Nevanlinna characteristic function of $W(z)$ as

$$T(r, W) := m(r, W) + N(r, W).$$

Definition 2 Let $W = W(z)$ be a k-valued algebroid function defined by (1) in $|z| < \infty$. The order ρ and lower order λ of $W(z)$ are defined respectively by

$$\rho = \limsup_{r \to \infty} \frac{\log^+ T(r, W)}{\log r}, \quad \lambda = \liminf_{r \to \infty} \frac{\log^+ T(r, W)}{\log r}.$$

Similarity, for the derivative $W'(z)$ of an algebroid function $W(z)$, we can define its order ρ' and lower order λ'.

In [1, 4], C. Chuang investigated the growth of meromorphic function $f(z)$ and its derivative $f'(z)$, and obtained the famous inequality as follows.

Theorem A Suppose that $f(z)$ is a meromorphic function in $|z| < \infty$, and $f(0) \neq \infty$, $f'(z)$ is its derivative, then there exists a positive constant r_0, such that for any $\tau > 1$ and $r > r_0$,

$$T(r, f) < C_\tau T(\tau r, f') + \log^+(\tau r) + 4 + \log^+ |f(0)|, \tag{3}$$