

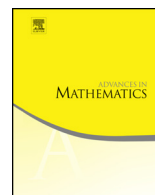


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## Cone-volume measure and stability <sup>☆</sup>



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### ABSTRACT

We prove stability results for two central inequalities involving the cone-volume measure of a centered convex body: the subspace concentration conditions and the U-functional/volume inequality.

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### 1. Introduction

Let  $\mathcal{K}^n$  be the set of all convex bodies in  $\mathbb{R}^n$  having non-empty interiors, i.e.,  $K \in \mathcal{K}^n$  is a convex compact subset of the  $n$ -dimensional Euclidean space  $\mathbb{R}^n$  with  $\text{int}(K) \neq \emptyset$ . As usual, we denote by  $\langle \cdot, \cdot \rangle$  the inner product on  $\mathbb{R}^n \times \mathbb{R}^n$  with associated Euclidean norm  $\| \cdot \|$ , and  $S^{n-1} \subset \mathbb{R}^n$  denotes the  $(n - 1)$ -dimensional unit sphere, i.e.,  $S^{n-1} = \{x \in \mathbb{R}^n : \|x\| = 1\}$ .

For  $K \in \mathcal{K}^n$  we write  $S_K(\cdot)$  and  $h_K(\cdot)$  to denote its surface area measure and support function, respectively, and  $\nu_K$  to denote the Gauß map assigning the outer unit normal  $\nu_K(x)$  to an  $x \in \partial_* K$ , where  $\partial_* K$  consists of all points in the boundary  $\partial K$  of  $K$  having a unique outer normal vector (cf. Section 2 for precise definitions). If the origin  $o$  lies in  $K \in \mathcal{K}^n$ , the *cone-volume measure* of  $K$  on  $S^{n-1}$  is given by

$$V_K(\omega) = \int_{\omega} \frac{h_K(u)}{n} dS_K(u) = \int_{\nu_K^{-1}(\omega)} \frac{\langle x, \nu_K(x) \rangle}{n} d\mathcal{H}_{n-1}(x), \tag{1.1}$$

where  $\omega \subseteq S^{n-1}$  is a Borel set and, in general,  $\mathcal{H}_k(x)$  denotes the  $k$ -dimensional Hausdorff measure. Instead of  $\mathcal{H}_n(\cdot)$ , we also write  $V(\cdot)$  for the  $n$ -dimensional volume.

The name cone-volume measure stems from the fact that if  $K$  is a polytope with facets  $F_1, \dots, F_m$  and corresponding outer unit normals  $u_1, \dots, u_m$ , then

$$V_K(\omega) = \sum_{i=1}^m V([o, F_i])\delta_{u_i}(\omega). \tag{1.2}$$

Here  $\delta_{u_i}$  is the Dirac delta measure on  $S^{n-1}$  concentrated at  $u_i$ , and for  $x_1, \dots, x_m \in \mathbb{R}^n$  and subsets  $S_1, \dots, S_l \subseteq \mathbb{R}^n$  we denote the convex hull of the set  $\{x_1, \dots, x_m, S_1, \dots, S_l\}$  by  $[x_1, \dots, x_m, S_1, \dots, S_l]$ . With this notation  $[o, F_i]$  is the cone with apex  $o$  and basis  $F_i$ .

In recent years, cone-volume measures have appeared and were studied in various contexts, see, e.g., F. Barthe, O. Guedon, S. Mendelson and A. Naor [3], K.J. Böröczky, E. Lutwak, D. Yang and G. Zhang [8,9], R. Gardner, D. Hug and W. Weil [20], M. Gromov and V.D. Milman [22], M. Ludwig [35], M. Ludwig and M. Reitzner [36], E. Lutwak, D. Yang and G. Zhang [41], A. Naor [44], A. Naor and D. Romik [45], G. Paouris and E. Werner [46], A. Stancu [51], G. Zhu [54,55], K.J. Böröczky and P. Hegedűs [5], L. Ma [43], Y. Huang, E. Lutwak, D. Yang and G. Zhang [32].

In particular, cone-volume measures are the subject of the *logarithmic Minkowski problem*. This is the particular interesting case  $p = 0$  of the general  $L_p$ -Minkowski problem which is at the core of the  $L_p$ -Brunn–Minkowski theory, one of the cornerstones of modern convex geometry. The  $L_p$ -Minkowski problem asks for a characterization of the  $L_p$  surface area measure

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