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Lagrangian Floer potential of orbifold spheres

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A B S T R A C T

For each sphere with three orbifold points, we construct an algorithm to compute the open Gromov–Witten potential, which serves as the quantum-corrected Landau–Ginzburg mirror and is an infinite series in general. This gives the first class of general-type geometries whose full potentials can be computed. As a consequence we obtain an enumerative meaning of mirror maps for elliptic curve quotients. Furthermore, we prove that the open Gromov–Witten potential is convergent, even in the general-type cases, and has an isolated singularity at the origin, which is an important ingredient of proving homological mirror symmetry.

Contents

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1. Introduction

Mirror symmetry reveals deep relations between symplectic and complex geometry. Closed-string mirror symmetry provides a powerful tool to enumerate rational curves by using complex deformation theory of the mirror, and open-string mirror symmetry builds a bridge between Lagrangian Floer theory in symplectic geometry and sheaf theory in complex geometry. Mirror symmetry has brought many exciting results to geometry in the last two decades.

Let $\mathbb{P}^1_{a,b,c}$ be an orbifold sphere with $a, b, c \geq 2$ equipped with the Kähler structure ω with constant curvature descended from its universal cover, which is a space-form (either the sphere, Euclidean plane, or hyperbolic plane). The mirror of $\mathbb{P}^1_{a,b,c}$ is a Landau–Ginzburg superpotential W, which is a holomorphic function defined over $\mathbb{C}^3$.

Closed-string mirror symmetry for $\mathbb{P}^1_{a,b,c}$ is a very rich subject and has been intensively studied. For instance, Frobenius structures and integrable systems of PDEs associated to W were constructed and used to understand the closed Gromov–Witten theory of $\mathbb{P}^1_{a,b,c}$ for $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} > 1$ by Milanov–Tseng [26] and Rossi [29] (and the general framework were studied in [34,13–15,20,11,12,16,30,36]). Explicit expressions of Saito's primitive forms [31,35,32,33] associated to W were studied and derived by Ishibashi–Shiraishi–Takahashi [21] and Li–Li–Saito [23]. Global mirror symmetry and LG/CY correspondence for $\mathbb{P}^1_{a,b,c}$ with $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$ were investigated and proved by Milanov–Ruan [24], Krawitz–Shen [22] and Milanov–Shen [25]. (There exist many literatures on various related topics, and here is just a partial list of them.) In the cases with $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} < 1$, the full superpotential W was not known but only the leading order terms $x^a + y^b + z^c - \sigma xyz$ (see [38,17]).

For open-string mirror symmetry, one direction of homological mirror symmetry conjecture was formulated and studied by Takahashi [40]. Roughly speaking it states that the derived category of coherent sheaves on $\mathbb{P}^1_{a,b,c}$ is equivalent to the derived Fukaya–Seidel category of the Landau–Ginzburg mirror. In the case of simple elliptic singularities (that is $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$) the conjecture was proved by Ueda [42]. The reversed direction of homological mirror symmetry, namely the equivalence between derived Fukaya category of $\mathbb{P}^1_{a,b,c}$ and the derived category of matrix factorizations on the Landau–Ginzburg mirror, was studied and derived in [10].