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Rates of mixing for the Weil–Petersson geodesic flow I: No rapid mixing in non-exceptional moduli spaces

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We show that the rate of mixing of the Weil–Petersson flow on non-exceptional (higher dimensional) moduli spaces of Riemann surfaces is at most polynomial.

Contents

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1. Introduction

The Weil–Petersson flow $\mathcal{WP}_t$ is the geodesic flow associated to the Weil–Petersson metric g_{WP} on the moduli spaces $\mathcal{M}(S) = \mathcal{M}_{g,n}$ of Riemann surfaces S of genus $g \geq 0$ and $n \geq 0$ punctures with $3g - 3 + n \geq 1$.

The Weil–Petersson (WP) metric is a natural object from the point of view of hyperbolic geometry of Riemann surfaces: for example, the WP metric has a simple expression in terms of the Fenchel–Nielsen coordinates (see Wolpert [10]), the growth of hyperbolic lengths of simple closed curves in Riemann surfaces is related to the volumes of moduli spaces equipped with the WP metric (see Mirzakhani [9]), and the thermodynamical invariants of the geodesics flows on Riemann surfaces allow the calculation of the Weil–Petersson metric (see McMullen [8]).

The Weil–Petersson flow is a dynamically interesting object: indeed, the WP metric is an incomplete negatively curved, Riemannian (and, in fact, Kähler) metric; in particular, the WP flow is an example of a singular hyperbolic flow.

The dynamics of the WP flow has the following properties:

- Wolpert [11] proved that the WP flow $\mathcal{WP}_t$ is defined for all times $t \in \mathbb{R}$ for almost every initial data with respect to the finite Liouville measure μ induced by the WP metric;
- Brock, Masur and Minsky [2] showed that the WP flow is transitive, its periodic orbits are dense and its topological entropy is infinite;
- Burns, Masur and Wilkinson [3] proved that the WP flow is ergodic and mixing (and even Bernoulli) non-uniformly hyperbolic flow whose metric entropy is positive and finite.

Intuitively, the ergodicity and mixing properties obtained by Burns, Masur and Wilkinson say that almost every WP flow orbit becomes equidistributed and all regions of the phase space are mixed together after a long time passes.

However, the arguments employed in [3] – notably involving Wolpert's curvature estimates for the WP metric, McMullen's primitive of the WP symplectic form, Katok and Strelcyn's work on Pesin theory for systems with singularities, and Hopf's argument for ergodicity of geodesic flows – do not provide rates of mixing for the WP flow. In other words, even though the WP flow mixes together all regions of the phase space after a certain time, it is not clear how long it takes for such a mixture to occur.

1.1. Main results

The goal of this paper is to prove that the rate of mixing of the WP flow is at most polynomial when the moduli space $\mathcal{M}_{g,n}$ is *non-exceptional*, i.e., $3g - 3 + n > 1$.