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Positive topological entropy and Δ-weakly mixing sets

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ABSTRACT

The notion of Δ-weakly mixing set is introduced and studied. It is proved that Δ-weakly mixing sets share many properties with weakly mixing sets, in particular, if a dynamical system has positive topological entropy, then the collection of Δ-weakly mixing sets is residual in the closure of the collection of entropy sets in the hyperspace. The existence of Δ-weakly mixing sets in a topological dynamical system admitting an ergodic invariant measure which is not measurable distal is obtained. These results generalize several well known results and also answer several open questions.

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1. Introduction

The main aim of this paper is to understand how complicated a topological dynamical system with positive topological entropy or with a non-distal ergodic invariant measure could be. To do so, we strengthen several known results and answer several open questions. To explain our results that we obtain, we start with some definitions.

A (topological) dynamical system (X, T) consists of a compact metric space (X, ρ) and a continuous map T from X to itself. We say that a dynamical system (X, T) is (topologically) transitive if for every two non-empty open subsets U and V of X there exists a positive integer n such that $U \cap T^{-n}V \neq \emptyset$. It is (topologically) weakly mixing if the product system $(X \times X, T \times T)$ is transitive. By the Furstenberg intersection Lemma, we know that (X, T) is weakly mixing, if and only if for every $n \geq 2$, the n-th product system $(X^n, T^{(n)}) := (X \times X \times \cdots \times X, T \times T \times \cdots \times T)$ (n-times) is transitive.

A dynamical system (X, T) is Δ-transitive if for every $d \geq 2$ there exists a residual subset X_0 of X such that for every $x \in X_0$ the diagonal d-tuple $x^{(d)} =: (x, x, \dots, x)$ has a dense orbit under the action of $T \times T^2 \times \cdots \times T^d$. There are two important classes of dynamical systems which are Δ-transitive. Glasner [13] proved that if a minimal system is weakly mixing then it is Δ-transitive. Recently, the authors of [23] observed that if a dynamical system admits a weakly mixing invariant measure with full support then it is also Δ-transitive.

In [27], Moothathu showed that Δ-transitivity implies weak mixing, but there exist strongly mixing systems which are not Δ-transitive. Using a class of Furstenberg families introduced in [8], the authors of [9] characterized the entering time sets of transitive points into open sets in Δ-transitive systems. We will give a more natural characterization of Δ-transitive systems by the generalized hitting time sets of open sets (see Proposition 3.1).

Following the intuition suggested by the definition of weak mixing, one can say that a dynamical system (X, T) is Δ-weakly mixing if the product system $(X \times X, T \times T)$ is Δ-transitive. But this definition is redundant as is in fact equivalent to Δ-transitivity (see Proposition 3.2), and hence Δ-transitivity has similar properties to weak mixing. Note however that the local notions of Δ-weakly mixing sets and Δ-transitive sets we will define below, are different (see Remark 3.5).

Blanchard and Huang [7] introduced a local version of weak mixing, named it as weak mixing sets. A closed subset A of X with at least two points is weakly mixing if for any $k \in \mathbb{N}$, any non-empty open subsets $U_1, U_2, \dots, U_k$ and $V_1, V_2, \dots, V_k$ of X intersecting A (that is $U_i \cap A \neq \emptyset$ and $V_i \cap A \neq \emptyset$) there exists $m \in \mathbb{N}$ such that $U_i \cap T^{-m}V_i$ is a non-empty open set intersecting A for each $1 \leq i \leq k$.

Inspired by this we say that a closed subset A of X is Δ-transitive if for every $d \geq 2$ there exists a residual subset A_0 of A such that for every $x \in A_0$, the orbit closure of the diagonal d-tuple $x^{(d)}$ under the action $T \times T^2 \times \cdots \times T^d$ contains A^d. A closed subset A of X with at least two points is Δ-weakly mixing if for every $n \geq 1$, A^n is Δ-transitive