

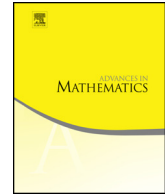


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On the conjecture \mathcal{O} of GGI for G/P



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ABSTRACT

In this paper, we show that general homogeneous manifolds G/P satisfy Conjecture \mathcal{O} of Galkin, Golyshev and Iritani which ‘underlies’ Gamma conjectures I and II of them. Our main tools are the quantum Chevalley formula for G/P and a theory on nonnegative matrices including Perron–Frobenius theorem.

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1. Introduction

Let X be a Fano manifold, i.e., a smooth projective variety whose anti-canonical line bundle is ample. The quantum cohomology ring $H^*(X, \mathbb{C})^1$ of X is a certain deformation of the classical cohomology ring $H^*(X, \mathbb{C})$ (§2.4 below). For $\sigma \in H^*(X, \mathbb{C})$, define the quantum multiplication operator $[\sigma]$ on $H^*(X, \mathbb{C})$ by $[\sigma](\tau) = \sigma \star \tau$ for $\tau \in H^*(X, \mathbb{C})$, where \star denotes the quantum product in $H^*(X, \mathbb{C})$. Let δ_0 be the absolute value of a maximal modulus eigenvalue of the operator $[c_1(X)]$, where $c_1(X)$ denotes the first Chern class of the tangent bundle of X . In [8], Galkin, Golyshev and Iritani say that X satisfies Conjecture \mathcal{O} if

- (1) δ_0 is an eigenvalue of $[c_1(X)]$.
- (2) The multiplicity of the eigenvalue δ_0 is one.
- (3) If δ is an eigenvalue of $[c_1(TX)]$ such that $|\delta| = \delta_0$, then $\delta = \delta_0 \xi$ for some r -th root of unity, where r is the Fano index of X .

In fact, in addition to Conjecture \mathcal{O} , Galkin, Golyshev and Iritani proposed two more conjectures called Gamma conjectures I, II, which can be stated under the Conjecture \mathcal{O} . Let us briefly introduce Gamma conjectures I, II in order to explain how it underlies them. Consider the quantum connection of Dubrovin

$$\nabla_{z\partial_z} = z \frac{\partial}{\partial z} - \frac{1}{z}(c_1(X)\star) + \mu,$$

acting on $H^*(X, \mathbb{C}) \otimes \mathbb{C}[z, z^{-1}]$, where μ is the grading operator on $H^*(X)$ defined by $\mu(\tau) = (k - \frac{\dim X}{2})\tau$ for $\tau \in H^{2k}(X, \mathbb{C})$. This has a regular singularity at $z = \infty$ and an irregular singularity at $z = 0$. Flat sections near $z = \infty$ can be constructed through flat sections near $z = 0$ classified by their exponential growth order, and they are put into correspondence with cohomology classes. To be precise, if X satisfies Conjecture \mathcal{O} , we can take a flat section $s_0(z)$ with the smallest asymptotics $\sim e^{-\delta_0/z}$ as $z \rightarrow +0$ along $\mathbb{R}_{>0}$. We transport $s_0(z)$ to $z = \infty$ and identify the corresponding class A_X called the principal asymptotic class of X . Then Gamma conjecture I states that the cohomology class A_X is equal to the Gamma class $\hat{\Gamma}_X$. Here $\hat{\Gamma}_X := \prod_{i=1}^n \Gamma(1 + \vartheta_i) \in H^*(X)$, where ϑ_i are the Chern roots of the tangent bundle TX for $i = 1, \dots, n$. Under further assumption of semisimplicity of the ring $H^*(X)$, we can identify cohomology classes A_δ corresponding to each eigenvalue δ in similar way. The classes A_δ form a basis of $H^*(X, \mathbb{C})$. Then Gamma conjecture II, a refinement of a part of Dubrovin’s conjecture [3], states that there is an exceptional collection $\{E_\delta \mid \delta \text{ eigenvalues of } [c_1(X)]\}$ of the derived category $D_{\text{coh}}^b(X)$ such that for each δ ,

¹ We use this notation for the quantum cohomology ring with the multiplication \star , and without quantum variables.

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