

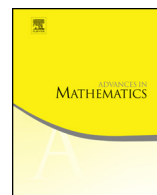


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Effective motives with and without transfers in characteristic p

Alberto Vezzani¹

Institut für Mathematik, Universität Zürich, Winterthurerstrasse 190, 8057 Zurich, Switzerland

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ABSTRACT

We prove the equivalence between the category $\mathbf{RigDM}_{\text{ét}}^{\text{eff}}(K, \mathbb{Q})$ of effective motives of rigid analytic varieties over a perfect complete non-archimedean field K and the category $\mathbf{RigDA}_{\text{Frobét}}^{\text{eff}}(K, \mathbb{Q})$ which is obtained by localizing the category of motives without transfers $\mathbf{RigDA}_{\text{ét}}^{\text{eff}}(K, \mathbb{Q})$ over purely inseparable maps. In particular, we obtain an equivalence between $\mathbf{RigDM}_{\text{ét}}^{\text{eff}}(K, \mathbb{Q})$ and $\mathbf{RigDA}_{\text{ét}}^{\text{eff}}(K, \mathbb{Q})$ in the characteristic 0 case and an equivalence between $\mathbf{DM}_{\text{ét}}^{\text{eff}}(K, \mathbb{Q})$ and $\mathbf{DA}_{\text{Frobét}}^{\text{eff}}(K, \mathbb{Q})$ of motives of algebraic varieties over a perfect field K . We also show a relative and a stable version of the main statement.

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Contents

1. Introduction	853
2. The Frob-topology	854
3. Rigid motives and Frob-motives	861
4. The equivalence between motives with and without transfers	870
Acknowledgments	878
References	878

E-mail address: vezzani@math.univ-paris13.fr.

¹ Current address: LAGA – Université Paris 13 (UMR 7539), 99 av. Jean-Baptiste Clément, 93430 Villetaneuse, France.

1. Introduction

Morel and Voevodsky in [21] introduced the derived category of effective motives over a base B which, in the abelian context with coefficients in a ring Λ and with respect to the étale topology, is denoted by $\mathbf{DA}_{\text{ét}}^{\text{eff}}(B, \Lambda)$. It is obtained as the homotopy category of the model category $\mathbf{Ch Psh}(\mathbf{Sm}/B, \Lambda)$ of complexes of presheaves of Λ -modules over the category of smooth varieties over B , after a localization with respect to étale-local maps (giving rise to étale descent in homology) and projection maps $\mathbb{A}_X^1 \rightarrow X$ (giving rise to the homotopy-invariance of homology). Voevodsky in [28,19] also defined the category of *motives with transfers* $\mathbf{DM}_{\text{ét}}^{\text{eff}}(B, \Lambda)$ using analogous constructions starting from the category $\mathbf{Ch PST}(\mathbf{Sm}/B, \Lambda)$ of complexes of presheaves *with transfers* over \mathbf{Sm}/B i.e. with extra functoriality with respect to maps which are finite and surjective. Both categories of motives can be stabilized, by formally inverting the Tate twist functor $\Lambda(1)$ in a model-categorical sense, giving rise to the categories of stable motives with and without transfers $\mathbf{DM}_{\text{ét}}(B, \Lambda)$ and $\mathbf{DA}_{\text{ét}}(B, \Lambda)$ respectively.

There exists a natural adjoint pair between the category of motives without and with transfers which is induced by the functor a_{tr} of “adjoining transfers” and its right adjoint o_{tr} of “forgetting transfers”. Different authors have proved interesting results on the comparison between the two categories $\mathbf{DA}_{\text{ét}}^{\text{eff}}(B, \Lambda)$ and $\mathbf{DM}_{\text{ét}}^{\text{eff}}(B, \Lambda)$ induced by this adjunction. Morel in [20] proved the equivalence between the stable categories $\mathbf{DA}_{\text{ét}}(B, \Lambda)$ and $\mathbf{DM}_{\text{ét}}(B, \Lambda)$ in case Λ is a \mathbb{Q} -algebra and B is the spectrum of a perfect field, by means of algebraic K -theory. Cisinski and Deglise in [8] generalized this fact to the case of a \mathbb{Q} -algebra Λ and a base B that is of finite dimension, noetherian, excellent and geometrically unibranch. Later, Ayoub (see [3, Theorem B.1]) gave a simplified proof of this equivalence for a normal basis B in characteristic 0 and a coefficient ring Λ over \mathbb{Q} that also works for the effective categories. In [2] the same author proved the equivalence between the stable categories of motives with and without transfers for a more general ring of coefficients Λ , under some technical assumptions on the base B (see [2, Theorem B.1]).

The purpose of this paper is to give a generalization of the effective result of Ayoub [3, Theorem B.1]. We prove an equivalence between the effective categories of motives with rational coefficients for a normal base B over a perfect field K of arbitrary characteristic. Admittedly, in order to reach this equivalence in characteristic p we need to consider a perfect base B^{Perf} and invert extra maps in $\mathbf{DA}_{\text{ét}}^{\text{eff}}(B^{\text{Perf}}, \mathbb{Q})$ namely the purely inseparable morphisms, or equivalently the relative Frobenius maps. This procedure can also be interpreted as a localization with respect to a finer topology, that we will call the Frobét-topology. The associated homotopy category will be denoted by $\mathbf{DA}_{\text{Frobét}}^{\text{eff}}(B^{\text{Perf}}, \mathbb{Q})$.

We remark that the approach *without transfers* is much more convenient when computing morphisms, and it is the most natural over a general base. On the other hand, Voevodsky proved a series of useful theorems for the category of motives *with transfers*

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