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Effective motives with and without transfers in characteristic p

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We prove the equivalence between the category $\mathbf{RigDM}^{\mathrm{eff}}_{\mathrm{\acute{e}t}}(K,\mathbb{Q})$ of effective motives of rigid analytic varieties over a perfect complete non-archimedean field K and the category $\mathbf{RigDA}^{\mathrm{eff}}_{\mathrm{Frob\acute{e}t}}(K,\mathbb{Q})$ which is obtained by localizing the category of motives without transfers $\mathbf{RigDA}^{\mathrm{eff}}_{\mathrm{\acute{e}t}}(K,\mathbb{Q})$ over purely inseparable maps. In particular, we obtain an equivalence between $\mathbf{RigDM}^{\mathrm{eff}}_{\mathrm{\acute{e}t}}(K,\mathbb{Q})$ and $\mathbf{RigDA}^{\mathrm{eff}}_{\mathrm{\acute{e}t}}(K,\mathbb{Q})$ in the characteristic 0 case and an equivalence between $\mathbf{DM}^{\mathrm{eff}}_{\mathrm{\acute{e}t}}(K,\mathbb{Q})$ and $\mathbf{DA}^{\mathrm{eff}}_{\mathrm{Frob\acute{e}t}}(K,\mathbb{Q})$ of motives of algebraic varieties over a perfect field K. We also show a relative and a stable version of the main statement.

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1. Introduction

Morel and Voevodsky in [21] introduced the derived category of effective motives over a base B which, in the abelian context with coefficients in a ring Λ and with respect to the étale topology, is denoted by $\mathbf{DA}^{\mathrm{eff}}_{\mathrm{ét}}(B,\Lambda)$. It is obtained as the homotopy category of the model category $\mathbf{Ch}\,\mathbf{Psh}(\mathrm{Sm}/B,\Lambda)$ of complexes of presheaves of Λ-modules over the category of smooth varieties over B, after a localization with respect to étale-local maps (giving rise to étale descent in homology) and projection maps $\mathbb{A}^1_X \to X$ (giving rise to the homotopy-invariance of homology). Voevodsky in [28,19] also defined the category of *motives with transfers* $\mathbf{DM}^{\mathrm{eff}}_{\mathrm{ét}}(B,\Lambda)$ using analogous constructions starting from the category $\mathbf{Ch}\,\mathbf{PST}(\mathrm{Sm}/B,\Lambda)$ of complexes of presheaves *with transfers* over Sm/B i.e. with extra functoriality with respect to maps which are finite and surjective. Both categories of motives can be stabilized, by formally inverting the Tate twist functor $\Lambda(1)$ in a model-categorical sense, giving rise to the categories of stable motives with and without transfers $\mathbf{DM}_{\mathrm{ét}}(B,\Lambda)$ and $\mathbf{DA}_{\mathrm{ét}}(B,\Lambda)$ respectively.

There exists a natural adjoint pair between the category of motives without and with transfers which is induced by the functor a_{tr} of "adjoining transfers" and its right adjoint o_{tr} of "forgetting transfers". Different authors have proved interesting results on the comparison between the two categories $\mathbf{DA}^{\mathrm{eff}}_{\mathrm{ét}}(B,\Lambda)$ and $\mathbf{DM}^{\mathrm{eff}}_{\mathrm{ét}}(B,\Lambda)$ induced by this adjunction. Morel in [20] proved the equivalence between the stable categories $\mathbf{DA}_{\mathrm{ét}}(B,\Lambda)$ and $\mathbf{DM}_{\mathrm{ét}}(B,\Lambda)$ in case Λ is a $\mathbb{Q}$-algebra and B is the spectrum of a perfect field, by means of algebraic K-theory. Cisinski and Deglise in [8] generalized this fact to the case of a $\mathbb{Q}$-algebra Λ and a base B that is of finite dimension, noetherian, excellent and geometrically unibranch. Later, Ayoub (see [3, Theorem B.1]) gave a simplified proof of this equivalence for a normal basis B in characteristic 0 and a coefficient ring Λ over $\mathbb{Q}$ that also works for the effective categories. In [2] the same author proved the equivalence between the stable categories of motives with and without transfers for a more general ring of coefficients Λ, under some technical assumptions on the base B (see [2, Theorem B.1]).

The purpose of this paper is to give a generalization of the effective result of Ayoub [3, Theorem B.1]. We prove an equivalence between the effective categories of motives with rational coefficients for a normal base B over a perfect field K of arbitrary characteristic. Admittedly, in order to reach this equivalence in characteristic p we need to consider a perfect base B^{Perf} and invert extra maps in $\mathbf{DA}^{\mathrm{eff}}_{\mathrm{ét}}(B^{\mathrm{Perf}},\mathbb{Q})$ namely the purely inseparable morphisms, or equivalently the relative Frobenius maps. This procedure can also be interpreted as a localization with respect to a finer topology, that we will call the Frobét-topology. The associated homotopy category will be denoted by $\mathbf{DA}^{\mathrm{eff}}_{\mathrm{Frobét}}(B^{\mathrm{Perf}},\mathbb{Q})$.

We remark that the approach *without transfers* is much more convenient when computing morphisms, and it is the most natural over a general base. On the other hand, Voevodsky proved a series of useful theorems for the category of motives *with transfers*