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Nearest neighbor Markov dynamics on Macdonald processes

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To the memory of Andrei Zelevinsky

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ABSTRACT

Macdonald processes are certain probability measures on two-dimensional arrays of interlacing particles introduced by Borodin and Corwin in [7]. They are defined in terms of nonnegative specializations of the Macdonald symmetric functions and depend on two parameters $q, t \in [0; 1)$. Our main result is a classification of continuous time, nearest neighbor Markov dynamics on the space of interlacing arrays that act nicely on Macdonald processes.

The classification unites known examples of such dynamics and also yields many new ones.

When $t = 0$, one dynamics leads to a new integrable interacting particle system on the one-dimensional lattice, which is a q-deformation of the PushTASEP (= long-range TASEP).

When $q = t$, the Macdonald processes become the Schur processes of Okounkov and Reshetikhin [41]. In this degeneration, we discover new Robinson–Schensted-type correspond-

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ences between words and pairs of Young tableaux that govern some of our dynamics.

Contents

1. Introduction

Since the end of 1990s there has been a significant progress in understanding the long time nonequilibrium behavior of certain *integrable* $(1+1)$-dimensional interacting particle systems and random growth models in the KPZ universality class. The miracle of integrability in most cases (with the notable exception of the partially asymmetric simple exclusion process) can be traced to an extension of the Markovian evolution to a suitable $(2+1)$-dimensional random growth model whose remarkable properties yield the solvability.

So far there have been two sources of such extensions. The first one originated from a classical combinatorial bijection known as the Robinson–Schensted–Knuth correspondence (RSK, for short). The RSK was first applied in this context by Johansson [25] and Baik–Deift–Johansson [1], and the dynamical perspective has been substantially developed by O'Connell [34–36], Biane–Bougerol–O'Connell [4] (see also Chhaibi [17]), Corwin–O'Connell–Seppäläinen–Zygouras [18], O'Connell–Pei [37], see also O'Connell–Seppäläinen–Zygouras [38].

The second approach was introduced by Borodin–Ferrari [11], and it was based on an idea of Diaconis–Fill [20] of extending intertwined "univariate" Markov chains to a "bivariate" Markov chains that projects to either of the initial ones. This approach was further developed in Borodin–Gorin [12], Borodin–Gorin–Rains [15], Borodin [5], Betea [2], and Borodin–Corwin [7]. In what follows we use the term *push-block dynamics* for the Markov chains constructed in this fashion (the reason for such a term will become clear later).

While the two resulting $(2+1)$-dimensional Markov processes that extend the same $(1+1)$-dimensional one share many properties — same fixed time marginals, same projections to many $(1+1)$-dimensional sections — the relation between them have so far