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The representation ring of the unitary groups and Markov processes of algebraic origin

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To the memory of Andrei Zelevinsky

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ABSTRACT

The paper consists of two parts. The first part introduces the representation ring for the family of compact unitary groups $U(1), U(2), \dots$. This novel object is a commutative graded algebra R with infinite-dimensional homogeneous components. It plays the role of the algebra of symmetric functions, which serves as the representation ring for the family of finite symmetric groups. The purpose of the first part is to elaborate on the basic definitions and prepare the ground for the construction of the second part of the paper.
The second part deals with a family of Markov processes on the dual object to the infinite-dimensional unitary group $U(\infty)$. These processes were defined in a joint work with Alexei Borodin (2012) [5]. The main result of the present paper consists in the derivation of an explicit expression for their infinitesimal generators. It is shown that the generators are implemented by certain second order partial differential operators with countably many variables, initially defined as operators on R.

Contents

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