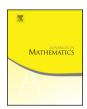


Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



Tensor diagrams and cluster algebras [☆]



Sergey Fomin a,*, Pavlo Pylyavskyy b

^a Department of Mathematics, University of Michigan, Ann Arbor, MI 48109, USA
^b Department of Mathematics, University of Minnesota, Minneapolis, MN 55414, USA

ARTICLE INFO

Article history: Received 12 December 2014 Accepted 5 April 2015 Available online 12 April 2016

To the memory of Andrei Zelevinsky

MSC: primary 13F60 secondary 05E99, 13A50, 15A72

Keywords: Cluster algebra Invariant theory Web basis Tensor diagram

ABSTRACT

The rings of $\operatorname{SL}(V)$ invariants of configurations of vectors and linear forms in a finite-dimensional complex vector space V were explicitly described by Hermann Weyl in the 1930s. We show that when V is 3-dimensional, each of these rings carries a natural cluster algebra structure (typically, many of them) whose cluster variables include Weyl's generators. We describe and explore these cluster structures using the combinatorial machinery of tensor diagrams. A key role is played by the web bases introduced by G. Kuperberg.

© 2016 Elsevier Inc. All rights reserved.

Contents

Introd	luction	718
Ackno	owledgments	722
Prelim	ninaries	723
1.	Tensors	723
2.	$\mathrm{SL}(V)$ invariants	725

E-mail addresses: fomin@umich.edu (S. Fomin), ppylyavs@umn.edu (P. Pylyavskyy).

 $^{^{\,\}pm}$ Partially supported by NSF grants DMS-1101152, DMS-1361789 (S.F.) and DMS-1068169, DMS-1351590 (P.P.).

^{*} Corresponding author.

3.	Cluster algebras	727
4.	Tensor diagrams	732
5.	Webs	739
6.	Special invariants	741
7.	Seeds associated to triangulations	746
Result	s and conjectures	751
8.	Main results	751
9.	Main conjectures	757
10.	Arborization	760
11.	Web gallery	764
Proofs	s	767
12.	Properties of special invariants	767
13.	Properties of special seeds	771
14.	Building a quiver	773
15.	Proof of the main theorem	776
16.	Other proofs	779
Apper	ndices	783
17.	Special choice of an initial seed	783
18.	Additional examples of seeds	785
Refere	ences	786

Introduction

Homogeneous coordinate rings of Grassmannians are among the most important examples of cluster algebras. Cluster structures in these rings [23,50] play a prominent role in applications of cluster theory arising in connection with integrable systems, algebraic Lie theory, Poisson geometry, Teichmüller theory, total positivity, and beyond; see, e.g., [21,24,25,29,32] and references therein. Within cluster algebra theory proper, Grassmannians provide the most concrete and accessible examples of naturally defined cluster algebras of infinite mutation type.

Despite their importance, cluster structures on Grassmannians are not well understood at all, apart from a few special cases. Just a tiny subset of their cluster variables have been explicitly described; we do not know which quivers appear in their seeds; we do not understand the structure of their underlying cluster complexes; and so on.

Let $Gr_{k,N}$ denote the Grassmann manifold of k-subspaces in an N-dimensional complex vector space. The corresponding cluster algebra has finite type (i.e., has finitely many seeds) if and only if $(k-2)(N-k-2) \leq 3$. All of the problems mentioned above are open for any Grassmannian of infinite cluster type, so in particular for $k=3, N\geq 9$. (The case k=2 has been well understood since the early days of cluster algebras, see [16, Section 12.2].)

We advocate the point of view that many aspects of cluster structures on Grassmannians are best understood within a broader range of examples coming from classical invariant theory. Recall that the homogeneous coordinate ring of $Gr_{k,N}$ (with respect to a Plücker embedding) is isomorphic to the ring of SL(V) invariants of N-tuples of vectors in a k-dimensional complex vector space V. More general rings of SL(V) invariants of collections of vectors and linear forms have been thoroughly studied by classical invariant

Download English Version:

https://daneshyari.com/en/article/4665089

Download Persian Version:

https://daneshyari.com/article/4665089

<u>Daneshyari.com</u>