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Quotients of Banach algebras acting on L^p-spaces ☆

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A B S T R A C T

We show that the class of Banach algebras that can be isometrically represented on an L^p-space, for $p \neq 2$, is not closed under quotients. This answers a question asked by Le Merdy 20 years ago. Our methods are heavily reliant on our earlier study of Banach algebras generated by invertible isometries of L^p-spaces.

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1. Introduction

An operator algebra is a closed subalgebra of the algebra $\mathcal{B}(\mathcal{H})$ of bounded linear operators on a Hilbert space $\mathcal{H}$. If A is an operator algebra and $I \subseteq A$ is a closed,

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two-sided ideal, then the quotient Banach algebra A/I is again an operator algebra, that is, it can be isometrically represented on a Hilbert space. This classical result is due to Lumer and Bernard, although the commutative case (when A is a uniform algebra) was proved earlier by Cole.

In Problem 3.8 of [7], Christian Le Merdy raised the question of whether this result can be generalized to Banach algebras acting on L^p-spaces for $p \in [1, \infty)$. More precisely, if $\mathscr{E}$ is a class of Banach spaces, we say that a Banach algebra A is an *$\mathscr{E}$-operator algebra* if there exist a Banach space E in $\mathscr{E}$ and an isometric homomorphism $\varphi: A \to \mathcal{B}(E)$. Given $p \in [1, \infty)$, we consider the class L^p of L^p-spaces, the class SL^p of Banach spaces that are isometrically isomorphic to subspaces of L^p-spaces, and the class QSL^p of Banach spaces that are quotients of SL^p-spaces.

The Bernard–Cole–Lumer Theorem asserts that L^2-operator algebras are closed under quotients. In Corollary 3.2 of [7], Le Merdy showed that QSL^p-operator algebras are closed under quotients. In Corollary 1.5.2.3 of [6], Marius Junge showed the analogous result for SL^p-operator algebras. Since the classes QSL^2 and SL^2 both agree with L^2, the results of Le Merdy and Junge are generalizations of the Bernard–Cole–Lumer Theorem.

As the authors point out in [7] and [6], the arguments used there are not suitable to deal with the more natural class of L^p-operator algebras. Indeed, the question of whether L^p-operator algebras are closed under quotients, for $p \neq 2$, remained open for 20 years. The case $p = 1$ of this question has recently been answered negatively in Theorem 6.2 of [2], using a classical result of Malliavin on the failure of spectral synthesis for $\ell^1(\mathbb{Z})$. In this paper, we answer negatively the remaining cases of the question. (Even for $p = 1$, we construct new examples of quotients of $\ell^1(\mathbb{Z})$ which cannot be represented on an L^1-space. These are, in particular, semisimple, unlike those constructed in [2].)

For $p \in [1, \infty)$, we consider the algebra $F^p(\mathbb{Z})$ of p-pseudofunctions on $\mathbb{Z}$. This algebra was introduced in the early 70's by Herz in [5], who denoted it $PF_p(\mathbb{Z})$. Algebras of p-pseudofunctions (also for locally compact groups other than $\mathbb{Z}$) have been studied in a number of places: [8,11,2,3,1], just to list a few.

The algebra $F^p(\mathbb{Z})$ is a semisimple, commutative Banach algebra with spectrum S^1. Given an open set V in S^1, we let I_V denote the largest closed two-sided ideal of $F^p(\mathbb{Z})$ that is supported on V. For $p \in [1, \infty) \setminus \{2\}$, and V neither empty nor dense in S^1, we show that $F^p(\mathbb{Z})/I_V$ is not an L^p-operator algebra; see Theorem 2.5. In fact, we even show that there is no injective, contractive homomorphism with closed range from $F^p(\mathbb{Z})/I_V$ to the algebra of bounded linear operators on *any* L^q-space for $q \in [1, \infty)$; see Remark 2.6.

Given the recent attention received by L^p-operator algebras, deciding whether these are closed under quotients becomes more relevant and technically useful. For example, consider the L^p-analogs $\mathcal{O}_n^p$ of the Cuntz algebras; see [10]. These are all simple, and any contractive, non-zero representation of any of them on an L^p-space is automatically injective (in fact, isometric). For $p = 2$, these two properties are well-known to be equivalent. However, for $p \neq 2$, they are not, since quotients of L^p-operator algebras are not in general representable on L^p-spaces. These two properties of $\mathcal{O}_n^p$ therefore

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